

Note that the inductive impedance of the secondary loop reflects into the primary as a capacitive impedance having a reactive component of  $-j189$  ohms. The reflected impedance appears in series with the primary-loop self-impedance. The equivalent circuit that the generator sees is shown in Fig. 4-13b. The input impedance is

$$\begin{aligned} Z_{in} &= Z_{11} + Z_{ref} = 20 + j1,131 + 421 - j189 \\ &= 441 + j942 = 1,040 \angle 64.9^\circ \text{ ohms} \end{aligned}$$

and the rms input current is

$$I_1 = \frac{115 \angle 0^\circ \text{ volts}}{1.04 \angle 64.9^\circ \text{ kilohms}} = 110 \text{ ma} \angle -64.9^\circ$$

Suppose it were desired to bring  $I_1$  into phase with the applied voltage. In other words, we wish to eliminate the reactive component of  $Z_{in}$  and bring about

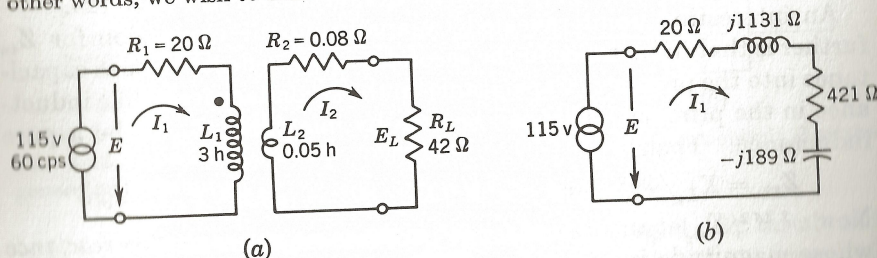


FIG. 4-13. (a) The input current ( $I_1$ ) is to be determined. (b) The equivalent circuit that the source looks into.

a condition of primary resonance. An easy way to do this would be to put a capacitor in series with the primary, so that its reactance cancels the inductive  $j942$ -ohm component of  $Z_{in}$ . Therefore,

$$X_C = 942 \text{ ohms} = \frac{1}{\omega C} \quad \text{and} \quad \omega = 377 \text{ radians/sec}$$

$$\begin{aligned} \text{Then } C &= \frac{1}{377 \times 942} = \frac{10^{-5}}{3.77 \times 0.942} = \frac{10^{-5}}{3.55} = 0.281 \times 10^{-5} \\ &= 2.81 \mu\text{f} \end{aligned}$$

**4-8. Transformer Transfer Impedance.** We have just completed an analysis which enables us to compute the impedance seen looking into a transformer. Once  $Z_{in}$  is evaluated, we can easily determine the input, or primary, current. What about the secondary current in a circuit such as Fig. 4-12a? To find  $I_2$ , we would set up the determinant based upon Eqs. (4-7) and (4-8):

$$I_2 = \frac{\begin{vmatrix} Z_{11} & E \\ Z_{12} & 0 \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = -\frac{EZ_{12}}{Z_{11}Z_{22} - Z_{12}^2}$$

The transfer impedance  $Z_{T_{12}}$  is therefore

$$Z_{T_{12}} = \frac{E}{I_2} = -\frac{Z_{11}Z_{22} - Z_{12}^2}{Z_{12}}$$

$$\text{where } Z_{11} = R_1 + j\omega L_1$$

$$Z_{12} = -j\omega M \text{ by the dot rule}$$

$$Z_{22} = R_2 + j\omega L_2$$

By looking at a circuit, we could then evaluate  $Z_{11}$ ,  $Z_{12}$ , and  $Z_{22}$ , from which  $Z_{T_{12}}$  is calculated. Once  $Z_{T_{12}}$  is known,  $I_2$  can be directly determined for any sinusoidal input on the primary side.

We have already shown that the input current is

$$I_1 = \frac{EZ_{22}}{Z_{11}Z_{22} - Z_{12}^2}$$

$$\text{Therefore, } \frac{I_2}{I_1} = \frac{-EZ_{12}/(Z_{11}Z_{22} - Z_{12}^2)}{EZ_{22}/(Z_{11}Z_{22} - Z_{12}^2)} = -\frac{Z_{12}}{Z_{22}}$$

or

$$I_2 = -\frac{Z_{12}}{Z_{22}} I_1$$

The relationship  $I_2 = -(Z_{12}/Z_{22})I_1$  is interesting in that it gives us a physical interpretation of the secondary loop. The numerator has the dimensions of ohms times amperes, or volts. The denominator has the dimensions of ohms and the numerator's volts divided by ohms gives the required dimension of amperes. We see then that the quantity  $-Z_{12}I_1$  can be interpreted as the voltage induced in the secondary winding by the current  $I_1$  in the primary. The secondary current  $I_2$  is equal to this induced voltage divided by the secondary loop self-impedance. Let us now evaluate  $I_2$  in Fig. 4-13a:

$$\begin{aligned} I_2 &= -\frac{Z_{12}}{Z_{22}} I_1 = -\frac{(-j\omega M)I_1}{Z_{22}} = \frac{(377)(0.39) \angle 90^\circ \text{ ohms} \times 110 \text{ ma} \angle -64.9^\circ}{46.1 \angle 24.1^\circ \text{ ohms}} \\ &= \frac{16.2 \angle 25.1^\circ \text{ volts}}{46.1 \angle 24.1^\circ \text{ ohms}} = 0.35 \angle 1^\circ \text{ amp} \end{aligned}$$

The phase angles of  $I_1$  and  $I_2$  are with respect to the input voltage, which was taken as our zero-degree reference. The load voltage is

$$E_L = I_2 R_L = (0.35 \angle 1^\circ \text{ amp})(42 \angle 0^\circ \text{ ohms}) = 14.7 \angle 1^\circ \text{ volts}$$