In essence, the dot rule says that if both current arrows flow into or out of the dots, the sign of $j\omega M$ is positive. If one current is directed in and the other out, $j\omega M$ is negative.

4-7. Transformer Input Impedance. When dealing with a transformer circuit in which the coupling is purely inductive, such as Fig. 4-12*a*, an error in the sign of M will not change the magnitude of the currents. The error will alter the phase of I_2 by only 180°.

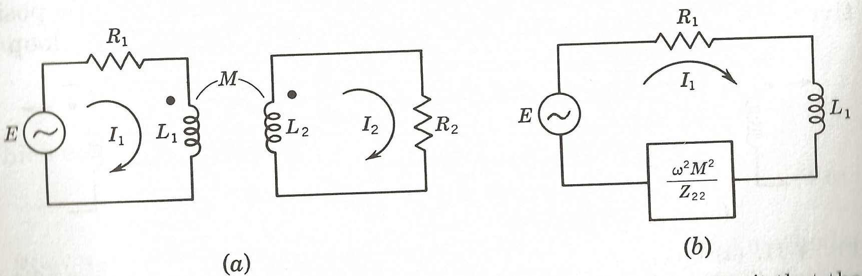

FIG. 4-12. (*a*) An inductively coupled circuit. (*b*) The equivalent circuit that the generator looks into.

If we were to write general loop equations for Fig. 4-12*a*, we should have

$$E = I_1 Z_{11} + I_2 Z_{12} \tag{4-7}$$

$$0 = I_1 Z_{12} + I_2 Z_{22} \tag{4-8}$$

The sign of Z_{12} (the mutual impedance) is negative, since I_1 flows into a dot and I_2 out. (Refer to dot rule.) If we solve for I_1,

$$I_1 = \frac{\begin{vmatrix} E & Z_{12} \\ 0 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{vmatrix}} = \frac{E Z_{22}}{Z_{11} Z_{22} - Z_{12}^2}$$

By dividing the numerator and denominator of the expression for I_1 by Z_{22}, we arrive at the form

$$I_1 = \frac{E}{Z_{in}} = \frac{E}{Z_{11} - Z_{12}^2/Z_{22}}$$

The input (driving point) impedance that the generator sees is then made up of two terms: Z_{11}, which is the self-impedance of loop 1, and $-Z_{12}^2/Z_{22}$, which is the reflected impedance of what is in loop 2 into loop 1. For an inductively coupled circuit, $Z_{12} = j\omega(\pm M)$. When M is negative, as in Fig. 4-12*a*, $Z_{12} = -j\omega M$. The reflected impedance is $Z_{12}^2/Z_{22} = -[j\omega(-M)]^2/Z_{22}$. It does not really matter if M is plus or minus when we compute Z_{in}, since M^2 is always positive. The

coefficient j when squared is $j^2 = -1$. From these considerations, the reflected impedance is therefore

$$-\frac{(-1)\omega^2 M^2}{Z_{22}} = +\frac{\omega^2 M^2}{Z_{22}}$$

The significance of this result is that no matter what the sign of M actually is, the input current I_1 can always be determined in a circuit such as Fig. 4-12*a* by computing the driving point impedance as

$$Z_{in} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}$$

Just *add* the reflected impedance term to the self-impedance of the primary loop, since they appear in series, as shown in Fig. 4-12*b*.

An interesting point revealed by analyzing the expression for Z_{in} further is that inductance in the secondary loop reflects back as capacitance into the primary. Capacitance in the secondary looks like inductance in the primary. This can be seen by imagining Z_{22} to be a pure inductance. Then

$$Z_{22} = X_{L_2}\underline{/90°} \quad \text{and} \quad \omega^2 M^2 / X_{L_2}\underline{/90°} = (\omega^2 M^2 / X_{L_2})\underline{/-90°}$$

Now $\omega^2 M^2 / X_{L_2}$ at an angle of $\underline{/-90°}$ is the same as a *capacitive reactance* whose magnitude is $\omega^2 M^2 / X_{L_2}$. In a similar manner, it can be shown that if the net reactance of the secondary is $Z_{22} = X_{C_2}\underline{/-90°}$, the reflected impedance will be inductive at an angle of $\underline{/90°}$. We can also see that when the secondary is open-circuited, or $Z_{22} = \infty$, the reflected impedance is zero. There is no secondary current to set up a flux which will thread the primary.

Let us try an illustrative problem involving these concepts.

Problem. A transformer has the following characteristics:

$L_1 = 3$ henrys	$L_2 = 0.05$ henry	$k \approx 1$
$R_1 = 20$ ohms	$R_2 = 0.08$ ohm	$E = 115$ volts, 60 cps

The generator source impedance is negligible, and the secondary load resistance (R_L) is 42 ohms. What is the primary current in Fig. 4-13*a*?

Solution

$$\omega = 2\pi f = 377 \text{ radians/sec}$$

$$M = k\sqrt{L_1 L_2} = \sqrt{0.15} = 0.39 \text{ henry}$$

$$Z_{12} = j\omega M$$

$$Z_{11} = R_1 + j\omega L_1 = 20 + (j377 \text{ ohms})(3 \text{ henrys})$$
$$= 20 + j1{,}131 \text{ ohms}$$

$$Z_{22} = 42.08 + (j377 \text{ ohms})(0.05 \text{ henry}) = 42.08 + j18.85$$
$$= 46.1\underline{/24.1°} \text{ ohms}$$

$$Z_{ref} = \frac{\omega^2 M^2}{Z_{22}} = \frac{(3.77)^2 \times 10^4 \times (0.39)^2}{46.1\underline{/24.1°}} = \frac{2.14 \times 10^4}{46.1\underline{/24.1°}}$$
$$= 0.0462 \times 10^4\underline{/-24.1°} = 462\underline{/-24.1°} = 421 - j189 \text{ ohms}$$