

There is no trick or shortcut in multiplying an $n \times n$ matrix with more than 3×3 entries. What is done is we simplify the process of multiplying, instead of doing it one way, we partition it. If we multiply an 8×8 by 8×1 matrix straight it would be so tedious that we need 8 rows with a column of 8 algebraic equation. You'll need a lot of paper for that matter.

In the example we now found this matrix,

$$\mathbf{AB} = \begin{vmatrix} A_{11}B_{11} + A_{12}B_{21} \\ A_{21}B_{11} + A_{22}B_{21} \end{vmatrix}$$

We need first to multiply the submatrices $A_{11} \times B_{11}$, $A_{12} \times B_{21}$,

$A_{21} \times B_{11}$, and $A_{22} \times B_{21}$. After you get the results add the matrices

$A_{11}B_{11} + A_{12}B_{21}$ and $A_{21}B_{11} + A_{22}B_{21}$. Note: ($A \times B$ is the same as AB). Then write the answer in matrix form above. That means $A_{11}B_{11} + A_{12}B_{21}$ is the first row with 4×1 entries and $A_{21}B_{11} + A_{22}B_{21}$ is the last row with also a 4×1 entries since there is only two rows in the form. In doing so, you arrive at an 8×1 entries.

Look at the partition in the previous note I'm assuming that you know basic matrix operations. In the figure, A_{11} , A_{12} , A_{21} , and A_{22} each has 4×4 entries. And B_{11} and B_{21} each has a 4×1 entries.