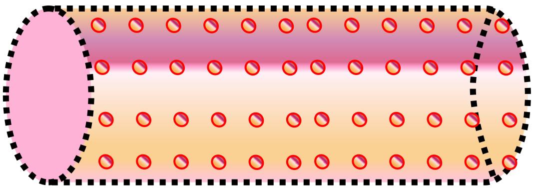
Current, Resistance

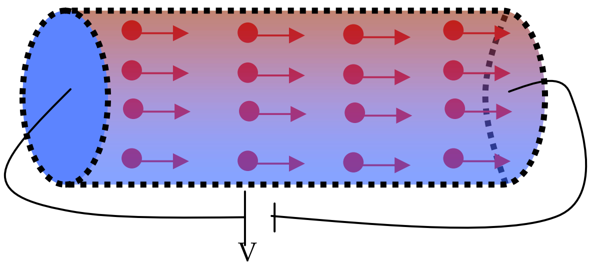
We will now look at the situation where charges are in motion - electrodynamics. The major difference between the static and dynamic cases is that $E = 0$ inside conductors for the static case, but $E \neq 0$ (i.e when a potential applied to the conductor) inside conductors for the dynamic case. And, if $E \neq 0$, then charges in the conductor feel a force ($F = qE$) and move in response to that force.

In the static case E=0, there is no charge inside the conductor. All charges are distributed over the surface of the conductor. There exists a *thermal* motion, and the *average position of the charges does not change* (even though electrons will be moving at approximately 10^5 m/s between collisions when the temperature is 300 K.)



In the dynamic case: $E \neq 0$:

When the electric field is present, the movement of the charges caused by the electric field is superimposed on the thermal motion, and there is a *net motion of the charges* (in the direction of the force exerted on the charges due to the electric field – in the same direction as the E-field for positive charges and in the opposite direction of the E-field for negative charges). The average speed at which the charges move is called the *drift velocity*, v_d. It is this flow of charge that we will study.



The motion of electrons (negative charge) in a particular direction can be replaced equivalently by the motion of positive charges in the opposite direction. In our discussion of charge motion we will deal primarily with motion of positive charges.

Any device that supplies the energy to cause the charges to separate is referred to as an EMF (electromotive force). The EMF produces the necessary electric field to cause the charges to move, e.g., battery (chemical energy), generator (mechanical energy), etc. The electric field produced by the battery causes the charges to move, and this motion of charges is called an *electric current*.

Electric current and current density

Imagine a section of the wire above, with a cross-sectional area A and with charges flowing with a velocity v_d . The direction of current flow is taken as the direction in which positive charges flow (even though in wires the negative charges, electrons, are the ones flowing).

The electric current is defined as the amount of charge crossing an imaginary boundary in the wire per unit time:

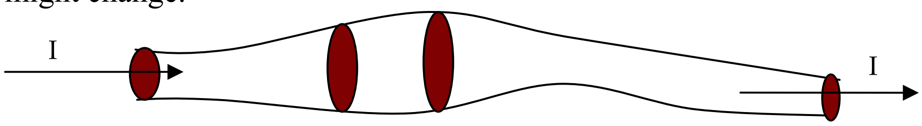
$$\text{electric current} = \frac{\text{amount of charge crossing a boundary}}{\textit{time}}$$

It is mathematically defines as

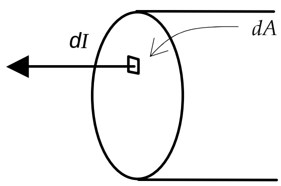
$$I = \frac{dQ}{dt}$$

The units of current are: $[\mathrm{I}] = \dfrac{[\mathrm{Q}]}{[\mathrm{t}]} = \dfrac{1\ \text{Coulomb}}{1\ \text{second}} \equiv 1\ \text{Ampere} = 1\ \text{Amp} = 1\text{A}$

The value of the electric current in a wire is the same no matter how the cross-sectional area of the wire might change.

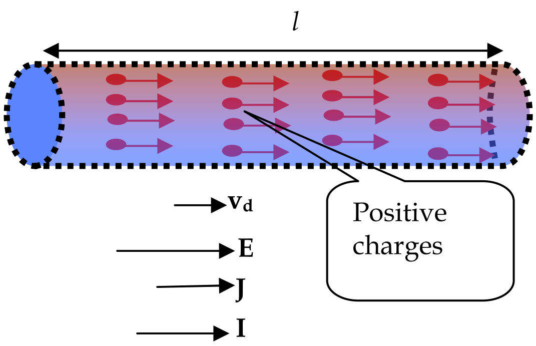


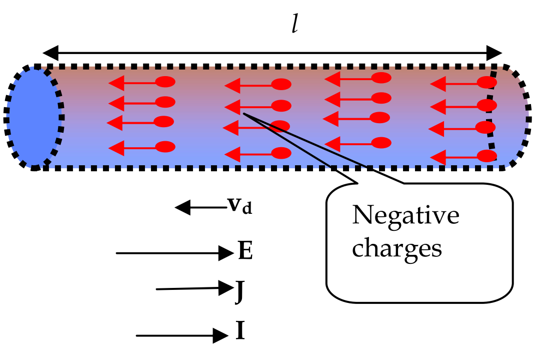
Another quantity that is closely related to current and does depend on the cross sectional area of the wire is current density J. The current density is the electric current per cross-sectional area, that is,



$$J = \frac{dI}{dA}$$

When the current density is constant: $J=I/A$.
Now let's ask the seemingly harmless question, how fast are the electrons going?





The speed of the electrons can be written as, $\upsilon = \frac{\ell}{t}$. The time can be found using the definition of current as in example 1,

$$I \equiv \frac{dQ}{dt} \Rightarrow \int_0^{Ne} dQ = I\int_0^t dt \Rightarrow Ne = It \Rightarrow t = \frac{Ne}{I}.$$

Now the speed becomes, $v_d = \frac{\ell I}{Ne} = \frac{\ell A}{N}\frac{I}{A}\frac{1}{e} = \frac{J}{ne}$

Define the free electron density and the current density,

$$\boxed{n \equiv \frac{N}{vol} \quad \text{The Definition of Free Electron Density}}$$

Example

What is the drift velocity of the electrons in a 2 mm diameter copper wire carrying a current of 1 A? Take n = 8.5 x 10^{28} electrons per cubic meter for copper. Before making the calculation, what is your guess?

$J = I/A = 1/(\pi 2\times10^{-3})^2 = 25330.3 A/m^2$;

$v_d = J/ne = 25330.3/(8.85*10^{28}*1.6*10^{-19}) = (1.8*10^{-6})$m/s

This velocity is so small that it is hard to understand why a light bulb goes on almost instantly when the switch is flipped. There is something we haven't accounted for in our model of charge flow in conductors.

Resistance and Resistivity

According to this model of current flow the charge will have a constant acceleration. Using Newton's Second Law, $\sum F = ma \Rightarrow a = \frac{F}{m} = \frac{qE}{m} = \frac{eV}{m\ell}$. If the acceleration is constant, the speed will increase indefinitely. This contradicts the concept of a drift velocity.

Because collisions take place as the electrons move through the wires, we can say that the wires are producing an impeding effect on the flow of those charges. This impeding effect is called electrical resistance.

We must include collisions between the electrons and atoms. If we call τ the average time between collisions, the average velocity of the electrons will roughly be $\bar{\upsilon} \approx a\tau$. Using the acceleration above and the equation for drift velocity,

$$\frac{j}{ne} = \frac{eV}{m\ell}\tau \Rightarrow \frac{I}{neA} = \frac{eV}{m\ell}\tau \Rightarrow V = I \cdot \frac{m}{\tau ne^2} \cdot \frac{\ell}{A}.$$

Define the resistivity as,

$$\boxed{\rho \equiv \frac{m}{\tau ne^2} \quad \text{The Definition of Resistivity}}$$

Notice that ρ is dependent on microscopic properties of the conducting material. The resistivity is difficult to calculate from these numbers, but it is not hard to measure. You will find tables of resistivity values. Sometimes the conductivity is tabulated. Conductivity, σ, is the reciprocal of the resistivity, $\sigma \equiv \frac{1}{\rho}$. Now, $V = I \cdot \rho \frac{\ell}{A}$. Define resistance as,

$$\boxed{R \equiv \rho \frac{\ell}{A} \quad \text{The Definition of Resistance}}$$

Resistance takes into account the geometrical properties of the material. Finally we have Ohm's Rule:

$$\boxed{V = IR \quad \text{Ohm's Rule}}$$

The units of resistance are: $[R] = \frac{[V]}{[I]} = \frac{1\ \text{Volt}}{1\ \text{Amp}} \equiv 1\ \text{Ohm} = 1\ \Omega$

Some values of resistivity, ρ, and conductivity, $\sigma = 1/\rho$.

copper:	$\rho = 1.7 \times 10^{-8}$ ohm m	$\sigma = 5.9 \times 10^{7}$ (ohm m)$^{-1}$
carbon:	$\rho = 3.5 \times 10^{-5}$ ohm m	$\sigma = 2.9 \times 10^{4}$ (ohm m)$^{-1}$
glass:	$\rho \sim 10^{12}$ ohm m	$\sigma \sim 10^{-12}$ (ohm m)$^{-1}$

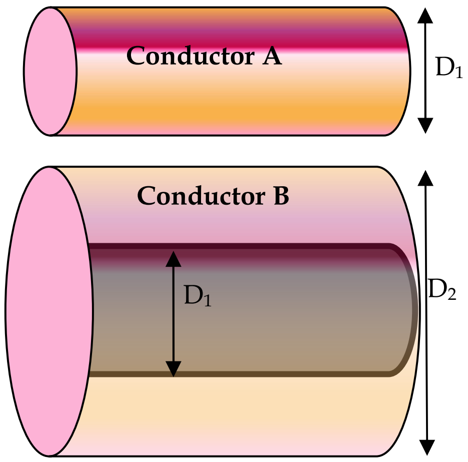
We notice that the resistivity depends on temperature. How does the resistivity of various types of materials change when the temperature is changed? Why? When the temperature of the material is increased then the charges in the material gains the energy. For the temperature dependence of the resistivity an approximate empirical expression can be written as:

$$\rho = \rho_0\left(1 + \alpha\left(T - T_0\right)\right)$$

here ρ_0 is resistivity of the material at the reference temperature T_0. The quantity α is the temperature coefficient of the resistivity.

Example

Two conductors of the same material and length have different resistances. Conductor A is a solid 1.00mm diameter wire. Conductor B is a tube of inner diameter 1.00mm and outer diameter 2.00mm. Find the ratio of the resistances of conductor A to conductor B.



From the definition of resistance, $R_A \equiv \rho \frac{\ell}{A_A}$ and

$R_B \equiv \rho \frac{\ell}{A_B}$.

The ratio is

$$\frac{R_A}{R_B} \equiv \frac{\rho \frac{\ell}{A_A}}{\rho \frac{\ell}{A_B}} = \frac{A_B}{A_A} = \frac{\frac{1}{4}\pi(D_2^2 - D_1^2)}{\frac{1}{4}\pi D_1^2} = \frac{2^2 - 1^2}{1^2} = \underline{\underline{3.00}}$$

Ohm's Law

Our aim here to understand why metals obey Ohm's Law. We know that macroscopic relation is given by

$$V = IR$$

without further discussion we use the relations $V=E.l$ and the relations between resistance and resistivity and definition of the current density we obtain

$$\vec{E} = \rho \vec{J}$$

We can say that at constant temperature the resistivity of the material is independent to the applied electric field **E**.(***Ohm's law, microscopic view***)

We can calculate drift speed interms of the applied field E. Fron Newton's second law:

$$a = \frac{eE}{m}$$

Consider an electron that has just undergone a typical effective collision. In the average time interval τ to the next collision, the electron will change its velocity in the direction of **–E** by an amount of $a\tau$:

$$v_d = a\tau = \frac{eE}{m}\tau$$

Then we can obtain another relation

$$v_d = \frac{eE}{m}\tau = \frac{J}{ne}$$

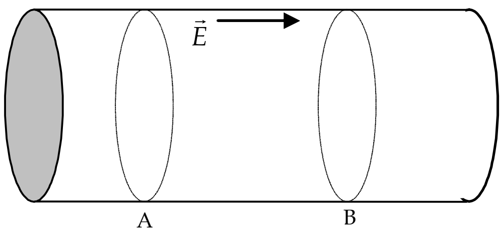
Remember $\vec{E} = \rho \vec{J}$, leads to

$$\rho = \frac{m}{ne^2\tau}$$

This is the microscopic Ohm's law. Note that τ is independent from electric filed.

Exercises

In electrical circuits, *E* and *J* are not measured, but *V* (voltage) and *I* (current) are. Look at the relationship between these quantities. Consider a length of the wire where an electric field *E* and the resulting current density *J* are present. Find the potential difference between points A and B, that is, find $V_{AB} = V_A - V_B$.

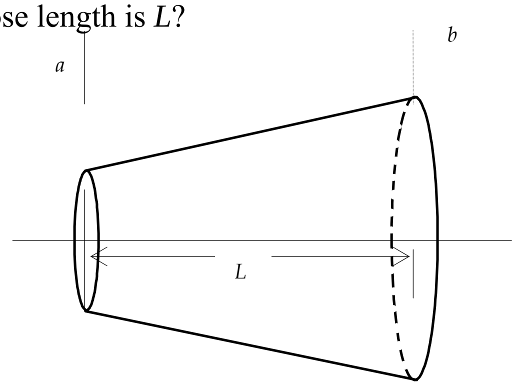


Define resistance. What is the unit of the resistance.

Take a 1 m length of copper wire with a diameter of 2 mm carrying a current of 2 A at a temperature of 20°C. What is the resistance of the wire, and what is the voltage between the ends of the wire?

Suppose the temperature of the above wire is increased to 220°C. What is the new resistance?

Can you find the resistance of a truncated cone made from a material whose resistivity is ρ, whose radii are *a* and *b*, and whose length is *L*?



Energy Transfer in Circuits

The work required to move a charge dq through a potential difference V is $dW = (dq)V$. Then, the power needed to accomplish this is

$$\text{Power} = P = \frac{dW}{dt} = \left(\frac{dq}{dt}\right)V, \text{ since } I = \left(\frac{dq}{dt}\right), \text{ Power } P = IV.$$

This is an expression that can be used for any circuit element. The units are amps.volts (A v) = Watt (W).

Using Ohm's Rule, power can be written without V or without I:

$$\boxed{\mathrm{P} = \mathrm{IV} = \frac{\mathrm{V}^2}{\mathrm{R}} = \mathrm{I}^2\mathrm{R} \qquad \text{Electric Power}}$$

Example

The extension cord of example 3 is connected to a 110V source. Find the (a)power supplied by the source, (b)power lost in the cord and (c)power supplied to the load.

(a)The electrical power supplied is $\mathrm{P} = \mathrm{IV} = (25.0\,\mathrm{A})(110\,\mathrm{V}) = \underline{\underline{2750\mathrm{W}}}$.

(b)The power lost in the cord can be found from the voltage drop,

$\mathrm{P} = \mathrm{IV} = (25.0\,\mathrm{A})(8.34\mathrm{V}) = \underline{\underline{209\mathrm{W}}}$.

The resistance of the cord could be used instead, $\mathrm{P} = \mathrm{I}^2\mathrm{R} = (25.0\mathrm{A})^2(0.334\Omega) = \underline{\underline{209\,\mathrm{W}}}$.

(c)By the Law of Conservation of Energy,

$\mathrm{P}_{\text{sup ply}} = \mathrm{P}_{\text{cord}} + \mathrm{P}_{\text{load}} \Rightarrow \mathrm{P}_{\text{load}} = \mathrm{P}_{\text{sup ply}} - \mathrm{P}_{\text{cord}} = 2750 - 209 = \underline{\underline{2540\mathrm{W}}}$.

Do you know why extension cords are rated for maximum length?

Explain why power lines use high voltage instead of high current.

Summary

The Definition of Current $I \equiv \frac{dQ}{dt}$

The Definition of Free Electron Density $n \equiv \frac{N}{vol}$

The Definition of Current Density $J = \frac{I}{A}$

Drift Velocity $J = nev_d$

The Definition of Resistivity $\rho \equiv \frac{m}{\tau n e^2}$

The definition of conductivity $\sigma = \frac{1}{\rho}$

The Definition of Resistance $R \equiv \rho \frac{\ell}{A}$

Ohm's Rule $V = IR$

Electric Power $P = IV = \frac{V^2}{R} = I^2 R$