

2-1 Thermal Noise in Resistors and Networks

As the name implies, *thermal noise* is due to the random motion of charge carriers in any conducting medium whose temperature is above absolute zero. The velocity of this motion increases with temperature in such a way that the electrical noise power density produced is proportional to the resistance of the conductor and to its absolute temperature, hence the name *thermal noise*. It is also called *white noise* because it has been shown both theoretically and experimentally to have a uniform spectrum up to frequencies on the order of 10^{13} Hz (just as white light is composed of all colors of the visible spectrum).

A metallic resistor may be considered a thermal noise source that can be represented by either of the noise equivalent circuits shown in Fig. 2-1. The *mean-square noise voltage* (V_n^2) and *current* (I_n^2) are given by the following expressions in which R is the resistance, $G = 1/R$ the conductance, T the temperature of the resistor in kelvin units, k Boltzmann's constant (1.38×10^{-23} J/K), and B the bandwidth in hertz in which the noise is observed.

$$V_n^2 = 4kTRB \quad (2-1)$$

$$I_n^2 = 4kTGB \quad (2-2)$$

(At low frequencies, practical resistors also exhibit excess current noise [4].)

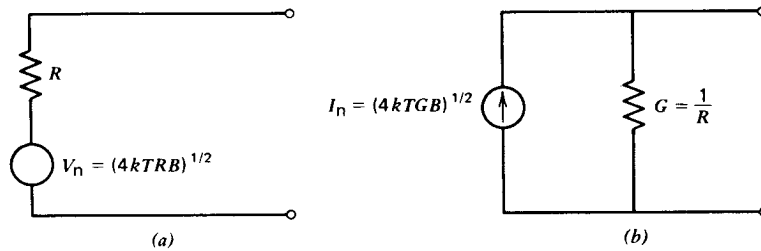
The noise power that is transmitted through a circuit is proportional to the circuit bandwidth. Consequently, the circuit bandwidth should never be greater than that necessary to transmit the desired signal if the maximum output signal-to-noise ratio (SNR) is to be achieved.

Example 2-1.1. Calculate the mean-square noise voltage produced in a 100-kilohm resistor in a bandwidth of 10^6 Hz at room temperature ($T = 20^\circ\text{C} = 293$ K).

$$4kT = 1.62 \times 10^{-20}$$

$$V_n^2 = 1.62 \times 10^{-20} \times 10^5 \times 10^6 = 16.2 \times 10^{-10} \text{ volts}^2$$

Fig. 2-1 Equivalent circuits to represent thermal noise in a resistor.



The rms noise voltage is

$$(V_n^2)^{1/2} = 40.3 \text{ microvolts}$$

If this 100-kilohm ($k\Omega$) resistor were in the input circuit of an electronic voltmeter that had a bandwidth of 1 MHz, no amount of gain built into the voltmeter would enable it to measure signals below 1 millivolt (mV) with accuracy.

Circuits containing more than one resistor may be analyzed by reducing them to one (Thévenin) equivalent resistance and applying (2-1) to obtain the mean-square noise voltage. The noise Thévenin equivalent of such a circuit is then a voltage source with this mean-square voltage in series with an ideal (noiseless) resistor equal in value to the Thévenin resistance. This is different from the signal Thévenin equivalent of the same circuit, as is made clear by Fig. 2-2, where a signal source driving a hypothetical noiseless load resistor R_i (representing the input of an amplifier) through three noisy resistors R_1 , R_2 ,

Fig. 2-2 (a) A resistive network driven by a voltage source; (b) Thévenin equivalent circuit for signal computation; and (c) Thévenin equivalent circuit for noise computation.

