

Let's suppose  $\tau_1 \gg \tau_2$

Then we can set  $\tau = \tau_1$

We can also write:

$$\tau_2 = \frac{\tau_1}{k} = \frac{\tau}{k}$$

with  $k \gg 1$

Substituting now in your equation:

$$V_{out} = V_{DD} \cdot \frac{\tau \cdot e^{-\frac{t}{\tau}} - \frac{\tau}{k} \cdot e^{-k \cdot \frac{t}{\tau}}}{\tau - \frac{\tau}{k}}$$

rearranging:

$$V_{out} = V_{DD} \cdot \frac{\tau \cdot e^{-\frac{t}{\tau}} \cdot \left(1 - \frac{1}{k} \cdot e^{-k}\right)}{\tau \cdot \left(1 - \frac{1}{k}\right)}$$

Now, if  $k \rightarrow \infty$  then  $1/k \rightarrow 0$  and  $e^{-k} \rightarrow 0$  then the terms in the brackets at both numerator and denominator  $\rightarrow 1$  from which:

$$V_{out} = V_{DD} \cdot e^{-\frac{t}{\tau}}$$

We will obtain the same result if  $\tau_2 \gg \tau_1$