

RC CIRCUIT

Introduction

A capacitor, often referred to as a condenser, is a simple electrical device consisting of two nearby conducting surfaces separated by an insulator. The capacitor can store charge of opposite sign on the two plates, and is of immense importance in the design of electronic devices. The purpose of this lab session is to examine how a capacitor stores its electric charge and how it discharges by producing a current through a resistor.

The ability of a capacitor to store charge was discovered accidentally in two independent experiments at different places during the same year. The first discovery was by Ewald Georg von Kleist in October of 1745 when he tried using an electrostatic generator to place a charge on an iron nail inside a small glass bottle. Later that same year, Anreas Cuneus, a lawyer who frequently visited one of the laboratories at the University of Leiden, was trying to electrify water. He used a chain hanging into a flask of water, and brought the end of the chain into contact with an electrostatic generator. In both cases, after disconnecting the generator, the experimenter touched the metal nail or chain inside the flask with one hand while the other hand still surrounded the outside of the container, and the experimenter experienced an electric shock as a result.

In both experiments the hand holding the container served as a conductor connected to a large reservoir for charge, namely the experimenter's own body, so his hand around the container acted as one of the plates of the capacitor, his body served as the ground, and the metal inside acted as the second plate separated from the first by the glass insulator. A net charge placed on the inner conductor produced Coulomb forces acting to induce a charge of opposite sign on the hand around the container, with the excess charge free to flow into or out of the ground. Removing the container from the generator left the metal inside and the experimenter's hand surrounding it outside with opposite net charge. But when the experimenter touched the inner conductor with his other hand, a sudden surge of charge could flow from one conductor to the other though the experimenter's body, with shocking results.

The second discovery, in Leiden, led to the earliest commonly-used capacitor, known thereafter as the “Leyden jar.” It consisted of a metal chain to conduct charge to a sheet of metal on the inside bottom of the jar, with the outside bottom surrounded by a second sheet of metal. The Leyden jar was widely used by early experimental investigators. No longer was it necessary to connect their experimental apparatus directly to the electrostatic generator. Charge could instead be placed in the jar, and carried to the experiment. Franklin, for example, used a Leyden jar to collect charge during a thunderstorm in his famous kite experiment to show that lightning was an electrical phenomenon.

The early ideas of the Leyden jar condensing the electrical fluid (or charge) of Franklin’s single fluid theory, and of using jars whose capacities would normally be measured in pints and quarts, is the origin of the commonly used terms “capacitor” and “condenser.”

The capacitors used in modern electronics are less cumbersome than the Leyden jar, though the principle of operation is the same. In a simple parallel plate capacitor (Fig. 1), two nearby conducting plates are separated by an insulator between them. If the two plates are connected to opposite terminals of a battery, one plate acquires a positive charge Q and the other a negative charge $-Q$. The electric field between the two plates, and therefore the potential difference between them, is proportional to the charge Q producing the field. The capacitance C , whose value depends on the detailed construction of the individual capacitor, is then defined to be the proportionality constant, $C=Q/V$, and its value states how much charge is

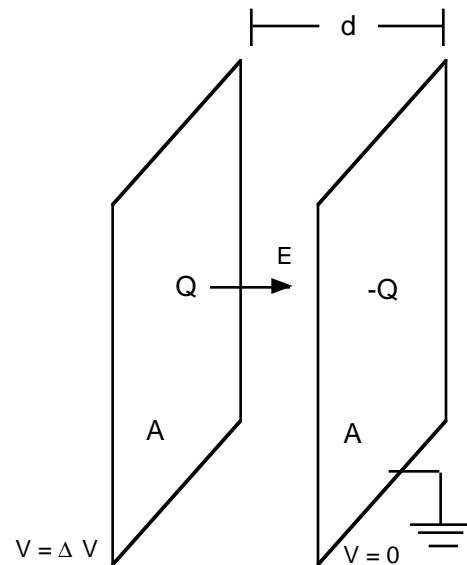


Figure 1
Parallel Plate Capacitor

stored per volt of applied potential. By convention, we take Q in this expression to be that at the positively charged plate, so that C is always positive. The unit of capacitance is the farad, named after Michael Faraday. The SI symbol for the farad is F, and 1 farad is equal to one volt per coulomb. It happens, however, that 1 F is a truly huge capacitance, and typical capacitors are more conveniently measured in micro-farads, with $1\ \mu\text{F} = 1 \times 10^{-6}\ \text{F}$, and sometimes even in pico-farads, with $1\ \text{pF} = 1 \times 10^{-9}\ \text{F}$. But because the commonly used abbreviation for “micro-farad” originated before present

conventions for the names and abbreviations of physical units were uniformly followed, the commonly encountered abbreviations “MF” and “mf” have also come to be used to mean “micro-farad,” rather than “milli-farad” as SI conventions would require.

In order to charge the capacitor, a battery or generator must move charge from one plate of the capacitor to the other through a potential difference, thereby doing work. Thus a capacitor not only stores opposite charge on its two plates, but also stores energy. As we move charge through the circuit to build up the potential difference across the capacitor, the potential difference through which the charge must be moved at each instant is

$$V(t) = \frac{Q(t)}{C}.$$

For any one specific capacitor, the energy stored in it should depend only on its final charge, not on the history of how the charge was built up. Therefore, to determine the energy stored, we can assume the charge was built up a constant rate starting from zero. Then V_{avg} , the average value of the potential over this time interval, is half the final V , and is therefore equal to $V/2 = Q/(2C)$. We would expect V_{avg} multiplied by the total Q that is passed from one plate to the other to give the work that was done, or the total energy stored, which is then

$$U = QV_{avg} = Q \frac{Q}{2C} = \frac{1}{2} CV^2. \quad (1)$$

The methods of integral calculus can be used to prove that the work done is indeed the charge multiplied by the time-average of the potential energy, provided the charge and therefore the potential energy difference, are built up at a constant rate.

Equation (1) itself can also be obtained directly by the methods of integral calculus. The work dW done to move a small charge dq through the circuit from one plate to another with potential difference $V(q)$ between them is $dW = Vdq$. Therefore, the work done to place total charge Q at voltage V on the capacitor is the sum of all the small increments of work, and the potential energy stored U is equal to the total work done, or

$$U = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 \quad (2)$$

in agreement with the result previously derived.

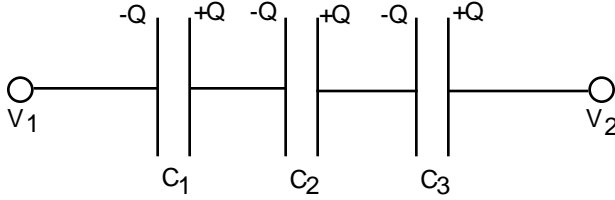


Figure 2 Capacitors in Series

Capacitors, like resistors, can be connected either in series (Fig. 2) or in parallel (Fig. 3), as well as in complex arrangements that can be broken down into combinations of parallel and series capacitors. For capacitors in series, the charge on

opposite plates is $\pm Q$ with the same Q for each capacitor. The voltage V_T across the series is the sum of the potential difference $V_i = Q/C_i$, across each, so that in Fig. 2

$$\begin{aligned} V_T &= V_1 + V_2 + V_3, \\ &= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \\ &= Q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) = \frac{Q}{C_T}. \end{aligned} \quad (3)$$

It can thus be seen that N capacitors in series have an overall capacitance (or “equivalent capacitance”) related by $C_T = Q/V_T$ to the voltage V_T between one battery terminal and the other, with the total capacitance given by

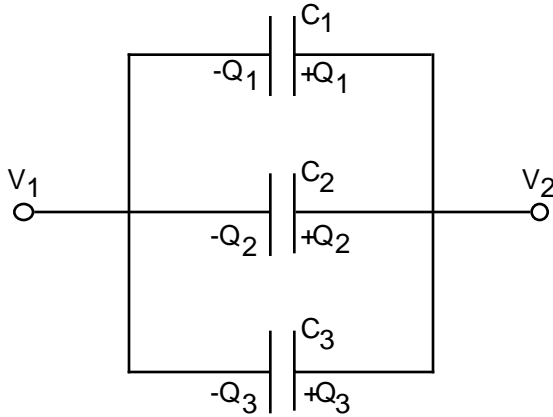


Figure 3 Capacitors in Parallel

$$\frac{1}{C_T} = \sum_{i=1}^N \frac{1}{C_i}. \quad (4)$$

For capacitors in parallel (Fig. 3), the voltage V across each is the same, but the

total charge stored $Q_T = C_T V$ is the sum of the charges $Q_i = C_i V$ for each, with

$$Q_T = Q_1 + Q_2 + Q_3, \quad (5)$$

which is then equal to

$$\begin{aligned} Q_T &= C_1 V + C_2 V + C_3 V \\ &= (C_1 + C_2 + C_3) V = C_T V. \end{aligned} \quad (6)$$

Thus the total charge stored is given in terms of an overall capacitance C_T by $Q = C_T V$ with

$$C_T = \sum_{i=1}^N C_i. \quad (7)$$

While the mathematical analysis for capacitors in series and in parallel is strikingly similar to the corresponding analysis of series and parallel resistors, the physical difference between the roles of capacitors and resistors in a circuit should always be kept in mind. The capacitor responds to an applied potential difference by storing charge on its plates that it can later discharge to produce a brief current in the direction opposite to that which charged it, while a resistor responds to an applied voltage by allowing a current to flow through it. The present experiment studies a combination of both behaviors by examining the short lived current that a capacitor produces when it discharges through a resistor.

The RC Circuit

Suppose a capacitor in the circuit shown in Fig. 4 has charge $\pm Q$ on its plates. At time $t = 0$ the switch is moved to position 2 and electrons from the negatively charged plate become free to flow through the resistor to the positively charged plate.

Charge conservation implies that the rate of charge leaving the capacitor to flow through the resistor is the charge per unit time, and is therefore the electric current $I(t)$ passing through

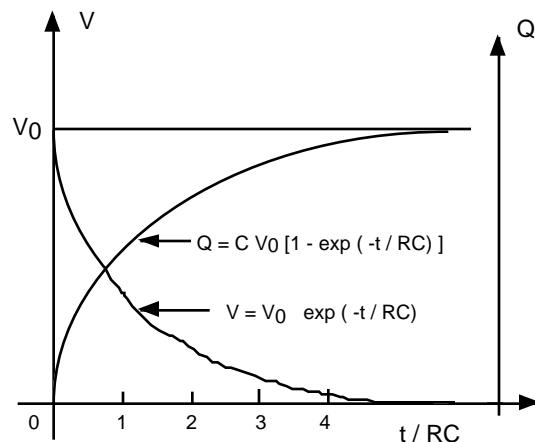
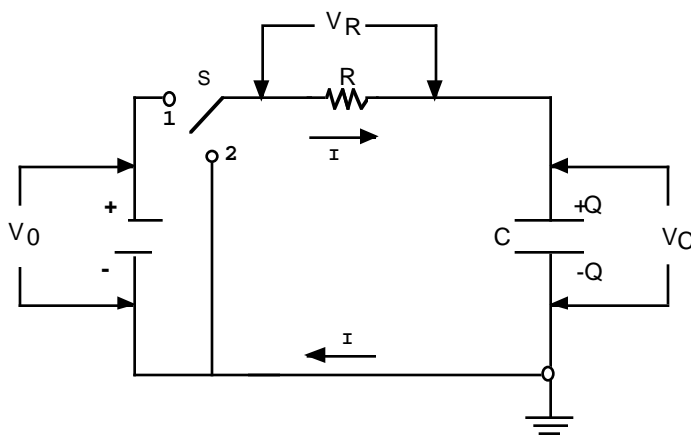


Figure 4 Charging and Discharging a Capacitor. Moving the switch in the top diagram to position 1 charges the capacitor with the time dependence for $q(t)$ shown in the bottom diagram, and next moving it to position 2 produces the time-dependent voltage shown.

the wire. According to Ohm's law, $I(t)$ is proportional to the voltage $V_R(t)$ across the resistor at each instant t , and is given by $I(t) = V_R(t)/R$. When the switch is in position 2, $V_R(t)$ equals $V_C(t)$ and is proportional to the charge still on the capacitor through the relation $V_C(t) = Q(t)/C$. Thus, we may conclude that the rate at which charge on the capacitor is being lost is the current $I(t)$ and is equal to $V_C(t)/R = Q(t)/(RC)$, and that the rate of decrease of Q is thereby proportional to Q itself.

In a great many different physical problems, as here, it happens that the rate of decrease of some physical quantity is proportional to its value still remaining. Such a relation holds also, for example, in radioactive decay.

As discussed in the Appendix, this condition implies that the quantity exhibits an exponentially decreasing dependence on time

$$Q(t) = Q(0)\exp(-kt) \quad (8)$$

in which k is the proportionality constant, in this case $1/(RC)$, between the quantity and its rate of decrease. Current, charge, and voltage are all found to follow the exponential decay shown in Fig. 4, as given by the expressions

$$Q(t) = Q(0)\exp\left(\frac{-t}{RC}\right) \quad (9)$$

$$V(t) = V(0)\exp\left(\frac{-t}{RC}\right). \quad (10)$$

$$I(t) = I(0)\exp\left(\frac{-t}{RC}\right). \quad (11)$$

If, instead of starting with the capacitor uncharged, we begin with $Q(0) = 0$ and connect the battery, the time dependence of the charge on the capacitor as noted in Fig. 4, is

$$Q(t) = CV_0(1 - e^{-t/RC}). \quad (12)$$

Note that the limiting value of the charge $Q(t)$ as $t \rightarrow \infty$ is $Q = C V_0$. As the capacitor becomes fully charged, the current approaches zero with the same time constant that applies when the capacitor is being discharged through the resistor.

The quantity $\tau \equiv 1/(RC)$ has dimensions of time and is referred to as the time constant for the circuit. It represents the time required for the charge to decay to a value equal to $1/e = 0.3679$ of its initial value, as true also for the voltage, and the current, as you can see by setting $\tau = RC$ in Eqs. (9) - (12). It is the quantity τ that we seek to measure in the experimental procedure.

Experiment: Finding the time constant of an RC circuit

The experiment uses a computer-based oscilloscope to observe the time dependence of the voltage $V_C(t)$ across the plates of an initially charged capacitor as the charge flows from one plate to the other through a resistance R .

The capacitor in the circuit illustrated in Fig. 4 was first charged up by connecting the capacitor to a battery, and then the switch S was used to let the charge flow through the resistor without the battery playing any further role. The charge on the capacitor, and therefore the voltage across it, according to the arguments presented in the previous sections, would then exhibit an exponential dependence on time.

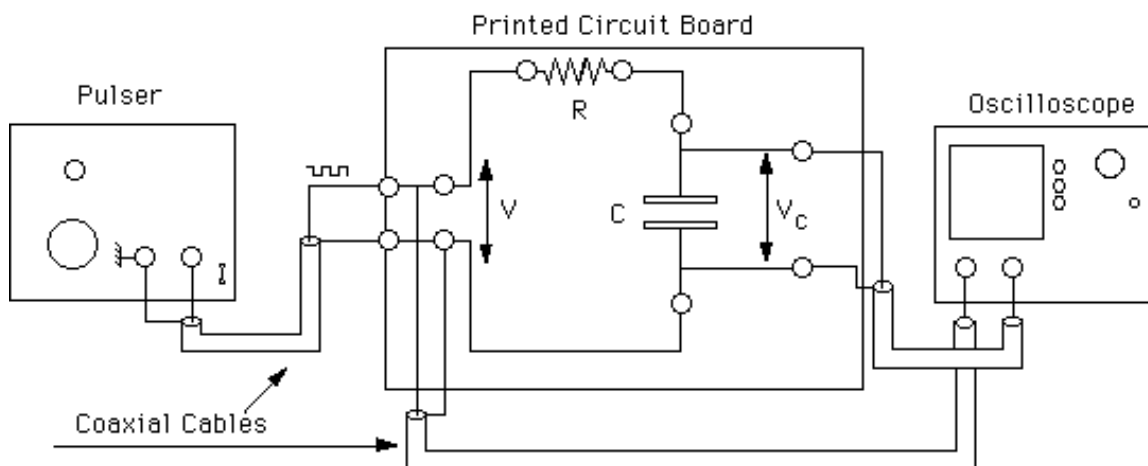


Figure 5 Pulse Generator Circuit

It proves more convenient to replace the switch by a square wave output of the pulse generator, using the circuit illustrated in Fig. 5. As illustrated in Fig. 6, the square-wave signal drives the system with a voltage that repeatedly switches abruptly back and forth between $+V_0$ and $-V_0$, as if the battery terminals to which the capacitor-resistor combination is connected were being interchanged repeatedly. Because the charge stored on the capacitor plates takes time to pass through the resistor, the capacitor does not respond by instantaneously showing the same $\pm V_0$ voltage variation. What results instead is a repeated exponential decay of voltage across the capacitor in response to the square-wave imposed voltage across the capacitor-resistor combination (Fig. 7). It should be remembered that voltages are always a difference between one potential and another, and that we could

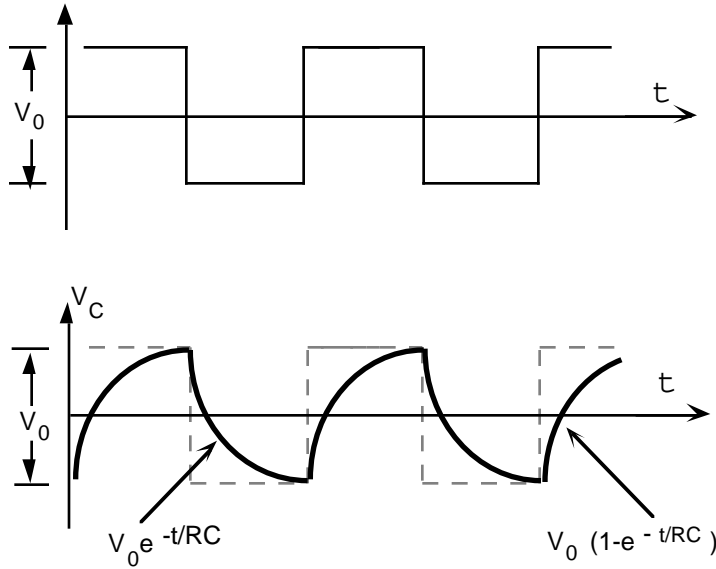


Figure 6
Square wave signal and response.

the time needed for a decrease of V_0 by a factor of $1/e$, to $0.3679 V_0$. Alternatively, by taking the logarithm of both sides of

$$V_C = V_0 \exp\left(\frac{-t}{RC}\right) = V_0 \exp\left(\frac{-t}{\tau}\right), \quad (13)$$

we find for $V_C(t)$

$$\ln V_C - \ln V_0 = \frac{-t}{RC}. \quad (14)$$

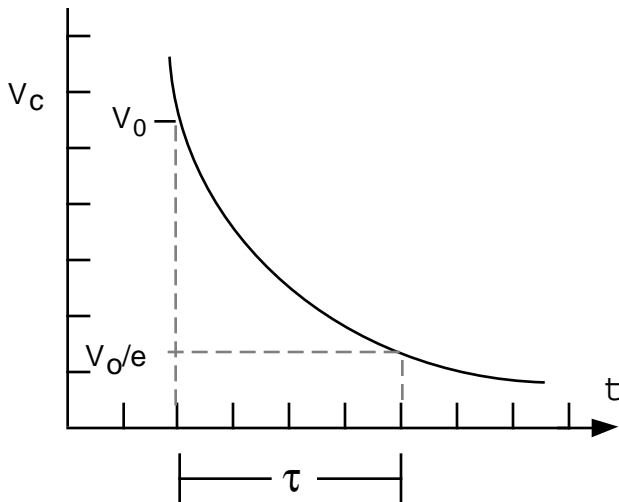


Figure 7
Voltage Decay Curve

regard the lower part of the square wave pattern to be at zero voltage, making the equations of the previous section directly applicable. The computer-based oscilloscope options allow adjusting the zero of the voltage scale to measure $V_C(t)$ relative to the lower voltage of the square-wave signal.

The time constant $\tau = RC$ can be found by measuring

Hence, we can also determine the time constant by measuring the slope of the $\ln V_C(t)$ vs. t curve.

Procedure

Connect your RC circuit to the pulse generator. The pulse generator should be set for square-wave output. Use one of the two input channels of the oscilloscope for the input square wave and the other for the square wave generator so that the two are superimposed on the display. The oscilloscope triggering should be set on “automatic” and “going down” with the triggering voltage not set to zero, but rather to about 0.5 V. The frequency of the pulse generator must be such that the corresponding period T is somewhat larger than the RC constant of the circuit. A good choice would be $T > 3\tau$. Once you are satisfied with your choice of frequency, display only the decaying part of V_C . Adjust the offset so that the decay curve is decaying to zero, as measured on the oscilloscope display grid.

Sketch in your lab book what you see on the oscilloscope.

Hit the “h” key to allow measuring the location of the data points by using the cross-hair cursor and reading the coordinates of each desired point at the bottom of the display. Find the distance on the time axis from the maximum voltage to the voltage divided by e , and thereby determine the time constant. Repeat this for other choices of voltage V_0 and the corresponding V_0/e from the same displayed data, obtaining five values of τ in all.

Use the computer to generate a plot of $\ln V_C$ vs. t for the stored data, and fit the plot by a straight line. The computer does this fit by choosing the line that minimizes the average square of the distance between the actual plot and the straight line based on sampling closely spaced times. Read, and record in your lab notebook, the deviation for the fit (equal to the square root of the average just described) as displayed numerically at the bottom of the computer display.

Compare both of the measured RC time constants to the one you calculated from the value of R and from the value of C printed on the capacitor.

The sample standard deviation for N measured data points whose values are x_1, x_2, \dots, x_N with average value \bar{x} is

$$s = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N-1}} . \quad (15)$$

When the experimental errors are purely random, s reflects the uncertainty in taking the representative value to be the average value \bar{x} of the N measured values, and the result of an experimental measurement of x would be expressed as $\bar{x} \pm s$. A broad distribution of measured values corresponds to a greater sample standard deviation, and to a greater uncertainty in how well the average value is likely to agree with the true value.

Compare the standard deviation for your measurements based on the five values of the time constant you measured by looking for pairs of voltages V_0 and V_0/e with the deviation recorded from the fit of $\ln V_C$ vs. t .

Appendix: The exponential decay law

We might imagine the time axis divided up into small time intervals each of short duration Δt , which may be regarded as intervals between rapid ticks of a clock. The elapsed time corresponds to a particular number N of such ticks, so that $t = N \Delta t$.

Now suppose that the rate at which a physical quantity Q decreases is proportional to $Q(t)$ itself. This means that in each time interval Q is reduced to a specific fraction of its value at the start of the interval. You might cleverly note at this point that if the logarithm of $Q(t)$ decreases by the same amount for each of these equally spaced time interval, this corresponds to the behavior described, in which the charge is reduced by the same factor over each such time interval. Thus, if we divide the time scale into very short intervals, the overall change in the quantity Q of interest is obtained by examining its logarithm and, for each of the N short time intervals elapsed, subtracting the same quantity from the logarithm of Q , meaning overall to subtract a quantity proportional to the overall time $N \Delta t$ elapsed since the initial time. Thus we expect

$$\ln Q(t) = \ln Q(0) - k(N \Delta t) = \ln Q(0) - kt$$

where the constant k characterizes the rate at which the value of Q decays. This leads immediately to

$$Q(t) = Q(0)e^{-kt}$$

which has the form of the exponential decay law, except that the constant k must be determined.

The calculus version of the derivation has the same physical content as that just described, and leads more easily to an evaluation of the parameter k . At any given time, the voltage across the capacitor (and across the resistor when the capacitor discharges through it) is

$$V_C(t) = \frac{Q(t)}{C} \tag{A-1}$$

where $\pm Q$ is the charge on the capacitor plates. The rate at which the charge Q leaves the capacitor is the current $I(t)$ through the circuit, which is related to the voltage $V_C(t)$ by Ohm's law, so that

$$-\frac{dQ}{dt} = I(t) = \frac{V_C(t)}{R} = \frac{Q(t)}{RC} . \quad (\text{A-2})$$

Thus

$$R \frac{dQ(t)}{dt} + \frac{Q(t)}{C} = 0 \quad (\text{A-3})$$

or, equivalently,

$$\frac{1}{Q} \frac{dQ(t)}{dt} = \frac{d}{dt} [\ln Q(t)] = -\frac{1}{RC} . \quad (\text{A-4})$$

Multiplying both sides by dt and integrating leads to

$$\int \frac{d}{dt} [\ln Q(t)] dt = - \int \frac{dt}{RC} \quad (\text{A-5})$$

or

$$\ln Q(t) = \ln c - \frac{t}{RC} \quad (\text{A-6})$$

where c is an additive constant to be determined. This implies

$$Q(t) = c \exp\left(\frac{-t}{RC}\right) \quad (\text{A-7})$$

and, because the initial charge at $t = 0$ is $Q(0)$, the constant c is equal to $Q(0)$ so that

$$Q(t) = Q(0) \exp\left(\frac{-t}{RC}\right) \quad (\text{A-8})$$

in agreement with Eq. (8). The proportionality between $V_C(t)$ and $Q(t)$ then also implies

$$V(t) = V(0) \exp\left(\frac{-t}{RC}\right). \quad (\text{A-9})$$

The relationship between the current and the voltage in Ohm's law then implies the similar exponential time dependence of the current

$$I(t) = I(0)\exp\left(\frac{-t}{RC}\right). \quad (\text{A-10})$$

If we begin with $Q(0) = 0$ and connect the battery, then Eq. (10) becomes instead $R\frac{dQ}{dt} + \frac{Q}{C} = V_0$. This yields, as noted in Fig. 4,

$$Q(t) = CV_0(1 - e^{-t/RC}). \quad (19)$$

Note that the limiting value of the charge $Q(t)$ as $t \rightarrow \infty$ is $Q = C V_0$. As the capacitor becomes fully charged, the current approaches zero with the same time constant that applies when the capacitor is being discharged through the resistor.

QUESTIONS

The following list of questions is intended to help you prepare for this laboratory session. If you have read and understood this write-up, you should be able to answer most of these questions. Some of these questions may be asked in the quiz preceding the lab.

- Explain, directly in terms of the physical nature of capacitance and resistance rather than from the mathematical form of the formula for the time dependence of Q in the RC circuit, what effect increasing R would have on how long it takes for half the stored charge to be depleted, and why.
- What effect would decreasing the capacitance C have on the time it takes to deplete half the stored charge, and why?
- Is the total capacitance (i.e., the “equivalent capacitance”) of two given capacitors in series greater than, less than, or equal to their total capacitance when they are in parallel, or does the answer depend on the particular capacitance of each capacitor? Why?
- When two capacitors are connected in parallel, and the capacitance of one of them is increased, how does the overall capacitance of the circuit change? Why?
- When two capacitors are connected in series, and the capacitance of one of them is decreased, how does the overall capacitance change? Why?
- In the experiment, we first determine the time constant by finding the difference between a voltage V_0 and V_0/e . Does it matter which value of V_0 we choose for doing this? Why?
- How is the definition of the sample standard deviation of the five data points similar to the deviation to be read for the fit of the straight line to the $\ln V_C$ vs. t plot, and how is it different?