LOW COMPLEXITY ESSENTIALLY MAXIMUM LIKELIHOOD DECODING OF PERFECT SPACE-TIME BLOCK CODES

*S. D. Howard, S. Sirianunpiboon**

Defence Science and Technology Organisation,
PO Box 1500, Edinburgh 5111, Australia

A. R. Calderbank

Electrical Engineering and Mathematics,
Princeton University, Princeton, NJ, 08544

ABSTRACT

Perfect space-time block codes (STBCs) were first introduced by Oggier et al. to have full rate, full diversity and non-vanishing determinant. A maximum likelihood decoder based on the sphere decoder has been used for efficient decoding of perfect STBCs. However the worst-case complexity for the sphere decoder is an exhaustive search. In this paper we present a reduced complexity algorithm for 3×3 perfect STBC which gives essentially maximum likelihood (ML) performance and which can be extended to other perfect STBC. The algorithm is based on the conditional maximization of the likelihood function with respect to one of the set of signal points given another. There are a number of choices for which signal points to condition on and the underlying structure of the code guarantees that one of the choices is good with high probability. Furthermore, the approach can be integrated with the sphere decoding algorithm with worst case complexity corresponding exactly to that of our algorithm.

Index Terms— Perfect space-time codes, fast maximum likelihood decoding, sphere decoding,

1. INTRODUCTION

The STBC discovered by Alamouti [1] which uses two transmit antennas facilitates high data rate, reliability and low complexity ML decoding. Orthogonal STBCs (OSTBC) [2] were designed to use more than two transmit antennas. However, as the number of transmit antennas increases, the rate becomes less attractive. Oggier et al.[3] introduced perfect space-time block codes which satisfy all of the following criteria: full-rate, full-diversity, non-vanishing determinant, good shaping and uniform average transmitted energy per antenna. These codes are constructed for $2\times 2, 3\times 3, 4\times 4$ and 6×6 multiple-input multiple-output (MIMO) systems. An example of a 2×2 perfect STBC is the Golden code [4] which has been incorporated into the 802.16e standard. The conventional ML decoder for perfect STBCs with an N-QAM or N-HEX constellation is based on an implementation of sphere decoding. In the case of Golden code, it has been widely reported that the worst case decoding complexity grows with the fourth power of the signal constellation size [5, 6]. Although, there is no report on the decoding complexity for other perfect codes, it is expected that when the channel matrix is close to singular, the preprocessing stage of the sphere decoding algorithm will yield a plane of possibilities rather than a single initial estimate. When this occurs, lattice point search degenerates to an exhaustive search.

In this paper we describe a fast decoding algorithm for perfect STBCs which give essentially ML performance with reduced complexity. The algorithm is based on the conditional maximum likelihood which is a technique widely used in statistical estimation and signal processing. The approach has been applied to the Golden code to obtain essentially ML performance with complexity $O(N^2)$ [7]. Here we show that the 3×3 perfect STBC can be decoded with complexity $O(N^6)$ to obtain essentially ML performance. Our approach can be applied to other perfect STBC to obtain low complexity decoding with essentially ML performance. Moreover, the approach can be integrated naturally with sphere decoding in a simple way leading to a sphere decoder with worst case complexity corresponding exactly to that of our algorithm.

*The work of S.Sirianunpiboon was supported by the Australian Defence Science and Technology Organisation (DSTO) Long Range Research Fellowship Program.

2. PERFECT SPACE-TIME BLOCK CODES

In this section we give the codeword matrices of the perfect STBCs for $3 \times 3, 4 \times 4$ MIMO systems in a form that will assist in the development of our algorithm.

2.1. 3×3 Perfect space-time block code

The perfect 3×3 STBC transmits nine complex (usually $N-$HEX constellation) information symbols $\{x_i\}_{i=1}^{9}$ over three time slots from three transmit antennas. The transmit codewords of the 3×3 perfect STBC can be expressed as

$$\mathbf{X}=B_1\begin{pmatrix} x_1 & x_2 & x_3 \\ jx_3 & x_1 & x_2 \\ jx_2 & jx_3 & x_1 \end{pmatrix}+B_2\begin{pmatrix} x_4 & x_5 & x_6 \\ jx_6 & x_4 & x_5 \\ jx_5 & jx_6 & x_4 \end{pmatrix}+B_3\begin{pmatrix} x_7 & x_8 & x_9 \\ jx_9 & x_7 & x_8 \\ jx_8 & jx_9 & x_7 \end{pmatrix} \quad (1)$$

978-1-4244-2354-5/09/$25.00

where the diagonal matrices B_i are

$$\begin{aligned} B_1 &= (1+j)I_3 + \Theta \\ B_2 &= (-1-2j)I_3 + j\Theta^2 \\ B_3 &= (-1-2j)I_3 + (1+j)\Theta + (1+j)\Theta^2 \end{aligned} \quad (2)$$

with $\Theta = diag(\theta_1, \theta_2, \theta_3), \theta_i = 2\cos(2^i\pi/7), j = e^{2\pi i/3}$ and I_n is the $n \times n$ identity matrix.

2.2. 4×4 Perfect space-time block code

The 4×4 perfect STBC transmits 16 complex ($N-$QAM constellation) information symbols $\{x_i\}_{i=1}^{16}$ over four time slots from four antennas. The codewords can be expressed as

$$\begin{aligned} \mathbf{X} =& B_1\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ ix_4 & x_1 & x_2 & x_3 \\ ix_3 & ix_4 & x_1 & x_2 \\ ix_2 & ix_3 & ix_4 & x_1 \end{pmatrix} + B_2\begin{pmatrix} x_5 & x_6 & x_7 & x_8 \\ ix_8 & x_5 & x_6 & x_7 \\ ix_7 & ix_8 & x_5 & x_6 \\ ix_6 & ix_7 & ix_8 & x_5 \end{pmatrix} \\ &+ B_3\begin{pmatrix} x_9 & x_{10} & x_{11} & x_{12} \\ ix_{12} & x_9 & x_{10} & x_{11} \\ ix_{11} & ix_{12} & x_9 & x_{10} \\ ix_{10} & ix_{11} & ix_{12} & x_9 \end{pmatrix} + B_4\begin{pmatrix} x_{13} & x_{14} & x_{15} & x_{16} \\ ix_{16} & x_{13} & x_{14} & x_{15} \\ ix_{15} & ix_{16} & x_{13} & x_{14} \\ ix_{14} & ix_{15} & ix_{16} & x_{13} \end{pmatrix} \end{aligned} \quad (3)$$

where

$$\begin{aligned} B_1 &= (1-3i)I_4 + i\Theta^2 \\ B_2 &= (1-3i)\Theta + i\Theta^3 \\ B_3 &= -iI_4 + (-3+4i)\Theta + (1-i)\Theta^3 \\ B_4 &= (-1+i)I_4 - 3\Theta + \Theta^2 + \Theta^3 \end{aligned} \quad (4)$$

with $\Theta = diag(\theta_1, \theta_2, \theta_3, \theta_4), \theta_i = 2\cos(2^i\pi/15)$.

3. FAST DECODING FOR THE 3×3 PERFECT STBC

In this section we show that fast decoding with reduced complexity and essentially ML performance can be achieved with a simple algorithm. We demonstrate this with the 3×3 perfect STBC which give the decoding complexity of $O(N^6)$, where N is the size of underlying HEX constellation. The approach is an extension of the ideas used to derive the fast optimal algorithm for multiplexing orthogonal design developed in [8], and applied to Golden code to obtain a fast essentially ML performance decoder [7].

Assume that the channel state information is available at the receiver. Let h_{ij} be the channel gain from transmit antenna i to a receive antenna j, the received signal is given by

$$\boldsymbol{r} = H\mathbf{X} + \boldsymbol{n} \quad (5)$$

where

$$H = \begin{pmatrix} h_{11} & h_{21} & h_{31} \\ h_{12} & h_{22} & h_{32} \\ h_{13} & h_{23} & h_{33} \end{pmatrix}.$$

Equation (5) can be rewritten as

$$\boldsymbol{r} = (x_1, x_2, x_3)\mathcal{H}_1 + (x_4, x_5, x_6)\mathcal{H}_2 + (x_7, x_8, x_9)\mathcal{H}_3 + \boldsymbol{n} \quad (6)$$

where$\boldsymbol{r} = (\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)$ contains the three received signal vectors $r_i = (r_{i1}, r_{i2}, r_{i3})$ with the component r_{ij} representing the received signal at antenna i in time slot j. The noise $\boldsymbol{n}$ is i.i.d Gausian noise with zero mean and covariance $2\sigma^2 I_3$ and

$$\mathcal{H}_1 = (\mathbf{H}_1, \mathbf{G}_1, \mathbf{C}_1), \mathcal{H}_2 = (\mathbf{H}_2, \mathbf{G}_2, \mathbf{C}_2), \mathcal{H}_3 = (\mathbf{H}_3, \mathbf{G}_3, \mathbf{C}_3),$$

where $\mathbf{H}_i, \mathbf{G}_i$ and $\mathbf{C}_i$ are induced channel matrices from the three transmit antennas to the first, second and the third receive antenna respectively. Explicitly

$$\mathbf{H}_i = \begin{pmatrix} b_{i1}h_{11} & b_{i2}h_{21} & b_{i3}h_{31} \\ jb_{i3}h_{31} & b_{i1}h_{11} & b_{i2}h_{21} \\ jb_{i2}h_{21} & jb_{i3}h_{31} & b_{i1}h_{11} \end{pmatrix}, \quad (7)$$

and similary for $\mathbf{G}_i$ and $\mathbf{C}_i$. The induced channel matrices $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ have the following property which is the basis of our fast decoding algorithm

$$\mathcal{H}_1\mathcal{H}_1^\dagger + \mathcal{H}_2\mathcal{H}_2^\dagger + \mathcal{H}_3\mathcal{H}_3^\dagger = 7\|H\|_F^2 I_3. \quad (8)$$

That is $\sum_{i=1}^K \mathcal{H}_i\mathcal{H}_i^\dagger$ is a multiple of identity. A similar property of the induced channel matrices holds for all of the perfect STBCs, including the Golden code. In fact, our fast decoding method will apply to any STBC with structure giving rise to a relation of the form (8).

Let $\boldsymbol{s} = (x_1, x_2, x_3), \boldsymbol{c} = (x_4, x_5, x_6), \boldsymbol{y} = (x_7, x_8, x_9)$, we can rewrite (6) as

$$\boldsymbol{r} = \boldsymbol{s}\mathcal{H}_1 + \boldsymbol{c}\mathcal{H}_2 + \boldsymbol{y}\mathcal{H}_3 + \boldsymbol{n}. \quad (9)$$

The likelihood function associated with (9) is

$$p(\boldsymbol{r}|\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{y},) \propto \exp\left(-\frac{1}{2\sigma^2}\|\boldsymbol{r} - \boldsymbol{s}\mathcal{H}_1 - \boldsymbol{c}\mathcal{H}_2 - \boldsymbol{y}\mathcal{H}_3\|^2\right). \quad (10)$$

Based on the conditional optimization described in [8], we first maximize (10) with respect to $\boldsymbol{c}$ and $\boldsymbol{y}$ given $\boldsymbol{s}$.

$$\begin{aligned} p(\boldsymbol{r}|\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{y}) \propto& \exp\left(-\frac{1}{2\sigma^2}\boldsymbol{r}'(I_9 - \mathcal{H}_1^\dagger(\mathcal{H}_1\mathcal{H}_1^\dagger)^{-1}\mathcal{H}_1)\boldsymbol{r}'^\dagger\right) \\ &\times \exp\left(-\frac{1}{2\sigma^2}(\boldsymbol{s} - \tilde{\boldsymbol{s}}(\boldsymbol{c}, \boldsymbol{y}))\mathcal{H}_1\mathcal{H}_1^\dagger(\boldsymbol{s} - \tilde{\boldsymbol{s}}(\boldsymbol{c}, \boldsymbol{y}))\right) \end{aligned}$$

where $\boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{c}\mathcal{H}_2 - \boldsymbol{y}\mathcal{H}_3$, and

$$\tilde{\boldsymbol{s}}(\boldsymbol{c}, \boldsymbol{y}) = (\boldsymbol{r} - \boldsymbol{c}\mathcal{H}_2 - \boldsymbol{y}\mathcal{H}_3)\mathcal{H}_1^\dagger(\mathcal{H}_1\mathcal{H}_1^\dagger)^{-1} \quad (11)$$

We now make what is essentially a zero forcing approximation, since $\mathcal{H}_1\mathcal{H}_1^\dagger$ is not generally a multiple of the identity. We take

$$\begin{aligned} \hat{\boldsymbol{s}}(\boldsymbol{c}, \boldsymbol{y}) &= \mathbf{Q}(\tilde{\boldsymbol{s}}(\boldsymbol{c}, \boldsymbol{y})) \\ &\equiv (Q(\tilde{x}_1(\boldsymbol{c}, \boldsymbol{y})), Q(\tilde{x}_2(\boldsymbol{c}, \boldsymbol{y})), Q(\tilde{x}_3(\boldsymbol{c}, \boldsymbol{y}))) \end{aligned} \quad (12)$$

where Q is the quantizer for the HEX constellation.

Substituting (12) into (10) we thus estimate $\boldsymbol{s}, \boldsymbol{c}$ and $\boldsymbol{y}$ as follows:

$$(\hat{\boldsymbol{c}}, \hat{\boldsymbol{y}}) = \arg\min_{\boldsymbol{c},\boldsymbol{y}\in\mathcal{C}} \|\boldsymbol{r} - \hat{\boldsymbol{s}}(\boldsymbol{c},\boldsymbol{y})\mathcal{H}_1 - \boldsymbol{c}\mathcal{H}_2 - \boldsymbol{y}\mathcal{H}_3\|^2, \quad (13)$$
$$\hat{\boldsymbol{s}} = \mathbf{Q}(\tilde{\boldsymbol{s}}(\hat{\boldsymbol{c}}, \hat{\boldsymbol{y}}))$$

where $\tilde{\boldsymbol{s}}(\boldsymbol{c},\boldsymbol{y})$ is given in (11).

If we first maximize (10) with respect to $\boldsymbol{s}$ and $\boldsymbol{y}$ given $\boldsymbol{c}$ we obtain the estimate

$$(\hat{\boldsymbol{s}}, \hat{\boldsymbol{y}}) = \arg\min_{\boldsymbol{s},\boldsymbol{y}\in\mathcal{C}} \|\boldsymbol{r} - \boldsymbol{s}\mathcal{H}_1 - \hat{\boldsymbol{c}}(\boldsymbol{s},\boldsymbol{y}) - \boldsymbol{y}\mathcal{H}_3\|^2, \quad (14)$$
$$\hat{\boldsymbol{c}} = \mathbf{Q}(\tilde{\boldsymbol{c}}(\hat{\boldsymbol{s}}, \hat{\boldsymbol{y}}))$$

where

$$\tilde{\boldsymbol{c}}(\boldsymbol{s},\boldsymbol{y}) = (\boldsymbol{r} - \boldsymbol{s}\mathcal{H}_1 - \boldsymbol{y}\mathcal{H}_3)\mathcal{H}_2^\dagger(\mathcal{H}_2\mathcal{H}_2^\dagger)^{-1} \quad (15)$$

Alternatively, if we maximize (10) with respect to $\boldsymbol{s}$ and $\boldsymbol{c}$ given $\boldsymbol{y}$ we obtain

$$(\hat{\boldsymbol{s}}, \hat{\boldsymbol{c}}) = \arg\min_{\boldsymbol{s},\boldsymbol{c}\in\mathcal{C}} \|\boldsymbol{r} - \boldsymbol{s}\mathcal{H}_1 - \boldsymbol{c}\mathcal{H}_2 - \hat{\boldsymbol{y}}(\boldsymbol{s},\boldsymbol{c})\mathcal{H}_3\|^2, \quad (16)$$
$$\hat{\boldsymbol{y}} = \mathbf{Q}(\tilde{\boldsymbol{y}}(\hat{\boldsymbol{s}}, \hat{\boldsymbol{c}}))$$

where

$$\tilde{\boldsymbol{y}}(\boldsymbol{s},\boldsymbol{c}) = (\boldsymbol{r} - \boldsymbol{s}\mathcal{H}_1 - \boldsymbol{c}\mathcal{H}_2)\mathcal{H}_3^\dagger(\mathcal{H}_3\mathcal{H}_3^\dagger)^{-1} \quad (17)$$

Equations (13), (14) and (16) each provide an algorithm for obtaining the estimate of $x_i, i = 1, \ldots, 9$, each of which involves at most N^6 evaluations of the right hand side of one of (13), (14) and (16). Now, we have three possible decoding solutions (13), (14) and (16). Of course if $\mathcal{H}_1\mathcal{H}_1^\dagger, \mathcal{H}_2\mathcal{H}_2^\dagger$ and $\mathcal{H}_3\mathcal{H}_3^\dagger$ were multiples of identity matrix, all of the optimizations (13), (14) and (16) would be exact ML and we would not need to make a choice. However, as we are making a zero forcing approximation, we need to choose the best alternative for each channel. One approach is to compute all three alternatives and take the alternative which maximizes the likelihood. The key to the current algorithm is that due to the structure of the code one of the three estimates is good, i.e., essentially ML, with very high probability.

The accuracy of the quantization depends on both the determinant (which determines signal to noise ratio) and condition number (which determines the accuracy of the zero forcing approximation) of $\mathcal{H}_1\mathcal{H}_1^\dagger, \mathcal{H}_2\mathcal{H}_2^\dagger$ or $\mathcal{H}_3\mathcal{H}_3^\dagger$. Fig.1 shows the distribution of min$(\gamma_1, \gamma_2, \gamma_3)$ where $\gamma_1, \gamma_2, \gamma_3$ represent the condition numbers. This shows that although the condition numbers can individually be large, the minimum of the three has a very high probability of being small.

We can reduce the computation by a factor of three by deciding on one of the three estimates based on the channel. A possible criterion is to choose to quantize the variables corresponding to the $\mathcal{H}_j$ with the largest value of det$(\mathcal{H}_j\mathcal{H}_j^\dagger)$.

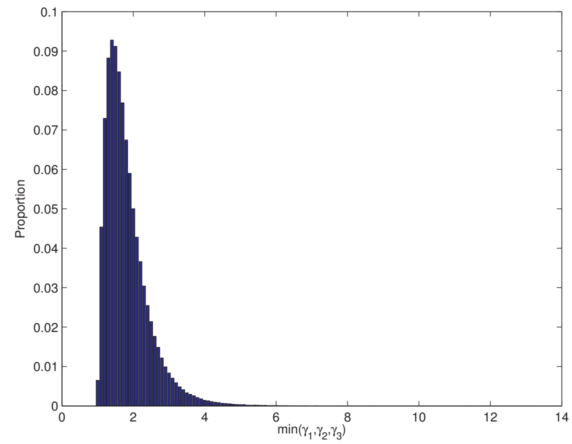

Fig. 1. Empirical distribution of min$(\gamma_1, \gamma_2, \gamma_3)$ for iid Gaussian channel coefficients.

Another choice is to quantize the variable for which the coreaponding matrix $\mathcal{H}_j\mathcal{H}_j^\dagger$ has the smallest condition number. For the Golden code these two criteria are equivalent, but here they are not. We have found experimentally that former criterion is just slightly better and obviates the need to compute all three estimates. The algorithm can be summarized as follows:

Let det$(\mathcal{H})$ = max[det$(\mathcal{H}_1\mathcal{H}_1^\dagger)$, det$(\mathcal{H}_2\mathcal{H}_2^\dagger)$, det$(\mathcal{H}_3\mathcal{H}_3^\dagger)$],

If det$(\mathcal{H}_1\mathcal{H}_1^\dagger)$ = det$(\mathcal{H})$

$$(\hat{\boldsymbol{c}}, \hat{\boldsymbol{y}}) = \arg\min_{\boldsymbol{c},\boldsymbol{y}\in\mathcal{C}} \|\boldsymbol{r} - \hat{\boldsymbol{s}}(\boldsymbol{c},\boldsymbol{y})\mathcal{H}_1 - \boldsymbol{c}\mathcal{H}_2 - \boldsymbol{y}\mathcal{H}_3\|^2$$
$$\hat{\boldsymbol{s}} = \mathbf{Q}(\tilde{\boldsymbol{s}}(\hat{\boldsymbol{c}}, \hat{\boldsymbol{y}}))$$

elseif det$(\mathcal{H}_2\mathcal{H}_2^\dagger)$ = det$(\mathcal{H})$

$$(\hat{\boldsymbol{s}}, \hat{\boldsymbol{y}}) = \arg\min_{\boldsymbol{s},\boldsymbol{y}\in\mathcal{C}} \|\boldsymbol{r} - \boldsymbol{s}\mathcal{H}_1 - \hat{\boldsymbol{c}}(\boldsymbol{s},\boldsymbol{y})\mathcal{H}_2 - \boldsymbol{y}\mathcal{H}_3\|^2$$
$$\hat{\boldsymbol{c}} = \mathbf{Q}(\tilde{\boldsymbol{c}}(\hat{\boldsymbol{s}}, \hat{\boldsymbol{y}}))$$

otherwise

$$(\hat{\boldsymbol{s}}, \hat{\boldsymbol{c}}) = \arg\min_{\boldsymbol{s},\boldsymbol{c}\in\mathcal{C}} \|\boldsymbol{r} - \boldsymbol{s}\mathcal{H}_1 - \boldsymbol{c}\mathcal{H}_2 - \hat{\boldsymbol{y}}(\boldsymbol{s},\boldsymbol{c})\mathcal{H}_3\|^2$$
$$\hat{\boldsymbol{y}} = \mathbf{Q}(\tilde{\boldsymbol{y}}(\hat{\boldsymbol{s}}, \hat{\boldsymbol{c}}))$$

where $\tilde{\boldsymbol{s}}, \tilde{\boldsymbol{c}}$ and $\tilde{\boldsymbol{y}}$ are given in (11), (15) and (17) respectively.

The perfect STBC are constructed in terms of information symbols either a QAM or HEX constellation. This means that the computational complexity of the quantization step is $O(1)$. Therefore our algorithm involves at most N^6 evaluations of likelihood function.

We compare the performance of the fast decoding algorithm described above with the ML decoder. The simulation was made for a Rayleigh fading channel model using 3×3 perfect STBC. Fig.2 shows the symbol error rate as a function of SNR using 4-HEX constellation. The result shows that our fast decoder is essentially ML decoder with complexity $O(N^6)$.

4. RELATIONSHIP TO SPHERE DECODING

The perfect STBC can be decoded with sphere decoder [3], but these suffer from the draw back that when the channel matrix is close to singular, the preprocessing stage of the sphere decoding algorithm yields a plane of possibilities rather than a single initial estimate. When this occurs, lattice point search degenerates to an exhuastive search with an overall complexity of order $O(N^9)$ (this becomes $O(N^{16})$ for 4×4 MIMO). In wireless communication, when the channel between base station and terminal is line of sight, the induced channel matrix is rank 1. The decoder has to be fabricated to handle worst case channel.

Having said that, our approach is, in fact, compatible with sphere decoding. The method of conditional maximization can be integrated with sphere decoding in a simple way leading to a sphere decoding algorithm with worst case complexity corresponding exactly to that of our algorithm. To see this, suppose we have determined that we only need to search symbols within the region $\mathcal{S} \subset \mathbb{C}^K$, defined by the equation

$$\mathcal{S}: \quad \boldsymbol{x}\boldsymbol{x}^\dagger < \rho, \quad \rho > 0. \tag{18}$$

Having decided on a sphere the reduced decoding problem becomes

$$\hat{\boldsymbol{x}} = \underset{\boldsymbol{x} \in \mathcal{C}^K \cap \mathcal{S}}{\arg\max}\, p(\boldsymbol{r}|\boldsymbol{x}). \tag{19}$$

Now consider the decoding problem

$$\boldsymbol{r} = \boldsymbol{x}\mathbf{H} + \boldsymbol{n} = \boldsymbol{x}_1\mathcal{H}_1 + \boldsymbol{x}_2\mathcal{H}_2 + \boldsymbol{n} \tag{20}$$

where we split the code vector $\boldsymbol{x}$ into two parts, $\boldsymbol{x}_1$ contains m symbols associated with m rows of $\mathbf{H}$ and $\boldsymbol{x}_2$ contains $K-m$ symbols associated with the $K-m$ rows of $\mathbf{H}$. Let $\boldsymbol{x}$ be a general vector in $\mathbb{C}^K$ and define $U_2 : \mathbb{C}^K \to \mathbb{C}^{K-m}$ by

$$\boldsymbol{x}U_2 = \boldsymbol{x}_2 \tag{21}$$

so that $\Pi_2 = U_2U_2^\dagger$ is the orthogonal projector on the "$\boldsymbol{x}_2$" subspace. Similary, we can define an orthogonal projection $\Pi_1 = U_1U_1^\dagger$ onto the "$\boldsymbol{x}_1$" subspace. If $\boldsymbol{x} \in \mathcal{S}$, then $\boldsymbol{x}_2$ contained in the region

$$\mathcal{S}_{\Pi_2} : \boldsymbol{x}_2\boldsymbol{x}_2^\dagger < \rho. \tag{22}$$

The optimization problem for $\boldsymbol{x}_2$ then reduces to

$$\hat{\boldsymbol{x}}_2 = \underset{\boldsymbol{x}_2 \in \mathcal{C}^{K-m} \cap \mathcal{S}_{\Pi_2}}{\arg\min} \|\boldsymbol{r} - \hat{\boldsymbol{x}}_1(\boldsymbol{x}_2)\mathcal{H}_1 - \boldsymbol{x}_2\mathcal{H}_2\|^2 \tag{23}$$

Hence the worst case complexity will be bound to $O(N^{K-m})$.

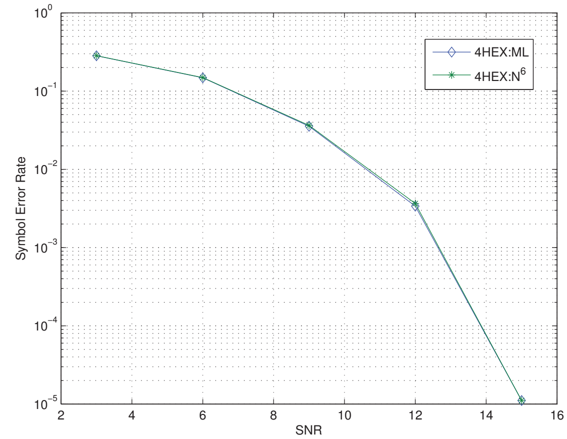

Fig. 2. Performance comparison between the low complexity decoder and the ML decoder

5. REFERENCES

[1] S. Alamouti, "Space-block coding: A simple transmitter diversity technique for wireless communications,," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, 1998.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[3] F. Oggier, G. Rekaya, J-C. Belfiore, and E. Viterbo, "Perfect space-time block codes," *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 3885–3902, Sept. 2006.

[4] J.-C Belfiore, G. Rekaya, and E. Viterbo, "The Golden code: A 2x2 full-rate space-time code with nonvanishing determinants," *IEEE Trans. Inform. Theory*, vol. 51, pp. 1432–1436, 2005.

[5] J. Paredes, A. B. Gershman, and M. G. Alkhanari, "A 2×2 space-time code with non-vanishing determinant and fast maximum likelihood decoding," in *Proc. IEEE ICASSP*, Apr 2007.

[6] S. Sezginer and H. Sari, "A full-rate full-diversity 2x2 space-time code for mobile WiMAX system," in *Proc. IEEE International Conference on Signal Processing and Communications*, Dubai, July 2007.

[7] S. Sirianunpiboon, A. R. Calderbank, and S. D. Howard, "Fast essentially maximum likelihood decoding of the golden code," *submitted to IEEE Trans. Information Theory*, 2008.

[8] S. Sirianunpiboon, Y. Wu, A. R. Calderbank, and S. D. Howard, "Fast optimal decoding of multiplexed orthogonal designs," *submitted to IEEE Trans. Info. Theory*, 2008.