

LOG PERIODIC FEEDS FOR LENS AND REFLECTORS

R. H. DuHamel and F. R. Ore
Collins Radio Company
Cedar Rapids, Iowa

Abstract

The application of unidirectional log periodic antennas as feeds for lens or reflectors to cover 10:1 or 20:1 bandwidths is described. Information on the primary patterns, phase center variation, input impedance, and aperture blocking of trapezoidal-tooth sheet structures is given so as to allow the design of feeds for a variety of lens and reflectors. Final results of pattern, gain and impedance measurements on a four-foot dish over the frequency range of 600 to 6000 mc are presented and a discussion of the slight sacrifice in gain to achieve this bandwidth is given.

Introduction

There are many applications in the communications, search, and ECM fields where it is quite desirable to have a high-gain antenna which will work over an extremely wide frequency range. Lens or reflector type antennas are often used, but their bandwidths have been limited by the primary feed. Ideally, the radiation pattern and input impedance of the primary radiator should be independent of frequency. The bandwidths of previous primary radiators have usually been on the order of 2 or 3:1. However, the recent discovery of log periodic^{1,2,3,4} and angular⁵ antennas with essentially frequency-independent operation over bandwidths of 10 or 20:1 provides the basis for new wide-band primary radiators.

This paper presents results of an investigation of the problems involved in the application of unidirectional log periodic feeds to reflector-type antennas. Sufficient information on the primary patterns, phase center, input impedance and aperture blocking of sheet trapezoidal-tooth log periodic structures is given to allow the antenna engineer to design feeds for a variety of lens and reflectors. Experimental results are given for a four-foot dish operating over the frequency range of 600 to 6000 mc.

Log Periodic Feed

Figure 1 is a sketch of a sheet trapezoidal-tooth log periodic antenna which will be considered in this paper as a feed. The angles α , β , and ψ define the extremities of the teeth, the tooth support section and the angle between the two half structures, respectively. The design ratio τ is defined as $\frac{R_{N+1}}{R_N}$. The ratio σ is set equal to $\sqrt{\tau}$. Since these antennas have been described previously, only a brief description of them will be given here.

The geometry of this type of structure is chosen so that the electrical properties must repeat periodically with the logarithm of the frequency. If the variation over one period is small, it is therefore small for all periods and the result is an extremely wide-band antenna. A period of frequency is defined by the range from τf to f . The antenna produces a

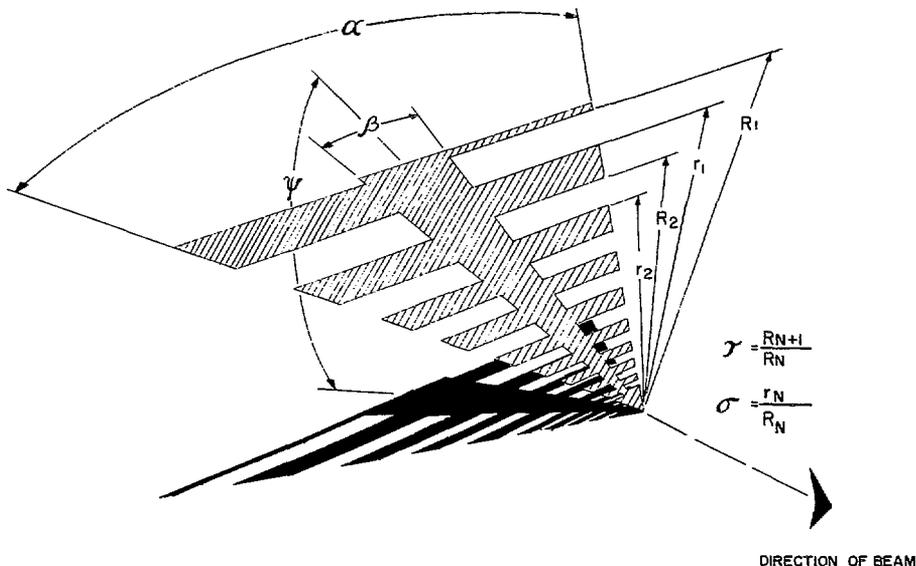


Fig. 1. Trapezoidal-tooth log periodic structure.

unidirectional, horizontally polarized beam pointing in the direction which the structure points. The two halves of the antenna are fed against each other either with a balanced line running between the two halves or an unbalanced line running along the center line of one half structure. The low-frequency cutoff occurs approximately when the longest tooth is $1/4$ wavelength long and the high-frequency cutoff occurs when the shortest tooth is somewhat less than $1/4$ wavelength long. The E- and H-plane patterns are the patterns in the xy and yz planes respectively. It is relatively easy to design these antennas to operate over 10:1 or greater bandwidths with essentially frequency-independent radiation patterns and input impedance.

General Feed Requirements

Obviously, for a very wide frequency range, the electrical characteristics of the feed should be essentially independent of frequency. The important electrical characteristics are the feed radiation pattern, input impedance, phase center, and the aperture blocking. The radiation patterns should be unidirectional and should have E-plane and H-plane beamwidths which give optimum gain for a given dish. These beamwidths will depend upon the shape of the dish, the F/D ratio and the desired illumination taper.

The requirements on the input impedance would depend upon the application, but in general, it may be said that the vswr should be at least less than 3:1. Of course, for radar applications, the vswr should be much less than this.

It is desirable that the feed look like a point source. Some feeds do not exhibit this property since the phase of the radiation pattern exhibits a complex behavior with radiation angle. The term point source

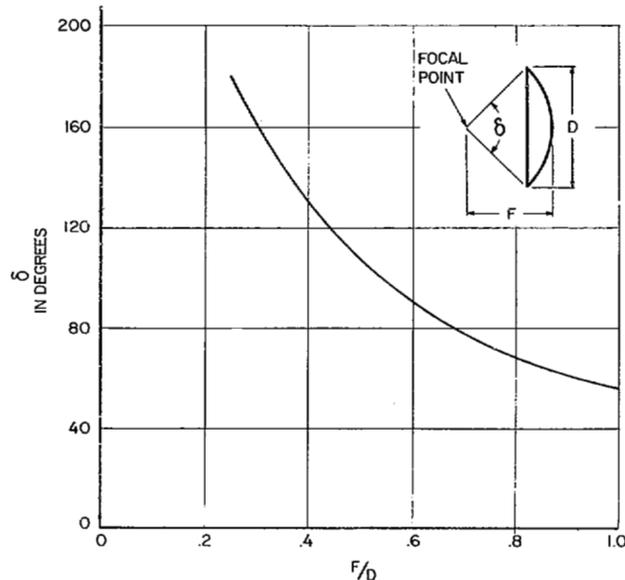


Fig. 2. Plot of angle δ subtended by a parabolic reflector vs. the F/D ratio.

cone is used to define the cone of radiation over which the feed behaves as a point source. In the text, the term "phase center" will also be used to refer to the equivalent point source. It is desirable that the E-plane and H-plane phase centers coincide.

Aperture blocking due to the feed can lead to increased secondary pattern side lobes and beamwidths. For extreme bandwidth applications using log periodic feeds, this can become a serious problem since the feed is many times the required size at the high end of the frequency range.

The ability of log periodic feeds to satisfy these feed requirements will be discussed in the following paragraphs.

Pattern Characteristics

Figure 2 is a curve of the angle, δ , subtended by a parabolic reflector as a function of the F/D ratio. In order to obtain high gain and low side lobes with a dish antenna, it is desirable to taper the aperture illumination. The optimum amount of taper⁶ is a rather insensitive function of the F/D ratio with an average value of about 9 to 10 db. Thus, for most cases, figure 2 can be used directly to determine the desired 10-db beamwidth of a feed for a given F/D ratio.

Sample voltage patterns of a trapezoidal-tooth log periodic antenna are given in figure 3 over a 10:1

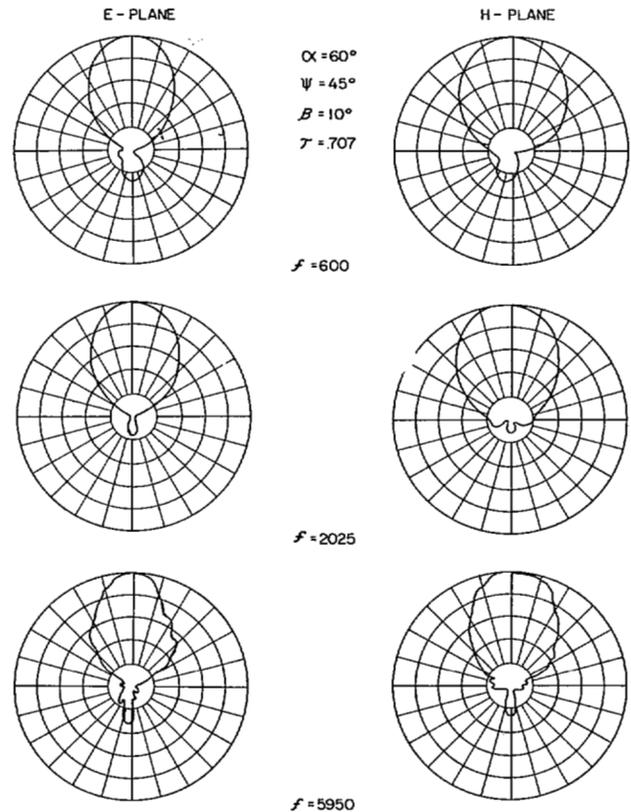


Fig. 3. Patterns of a logarithmically periodic feed over a 10:1 frequency range.

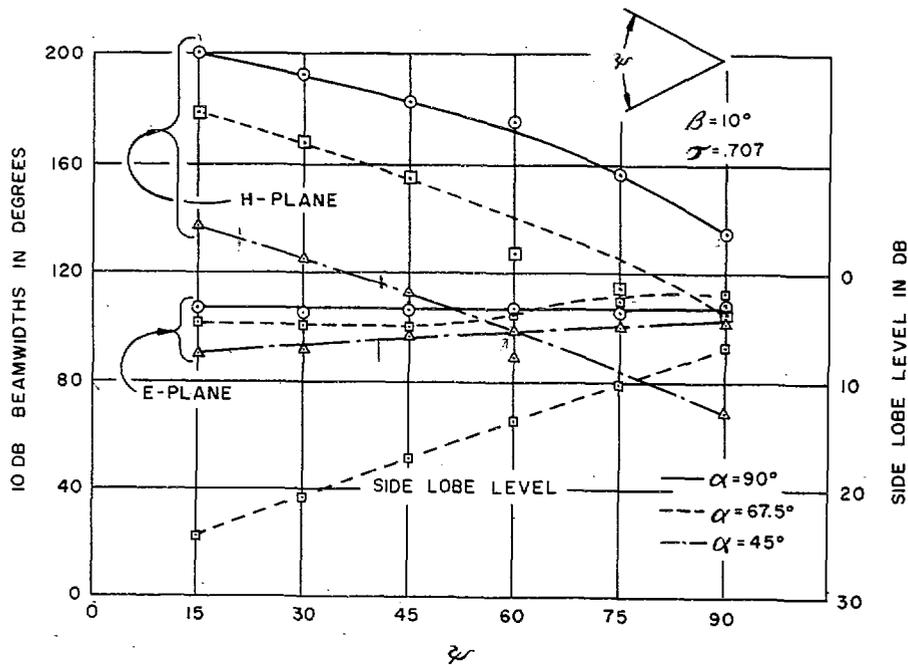


Fig. 4. E and H plane 10 db beamwidths and side lobe level as a function of ψ .

frequency range. These demonstrate the small variation of the beamwidths over the frequency range. The variation of the beamwidths is generally on the order of $\pm 8\%$.

The beamwidth and side-lobe level can be controlled to a considerable extent by the design parameters, α , β , ψ and τ . Figure 4 shows the variation of the

E- and H-plane beamwidths and side-lobe level as a function of ψ for several values of α . For these curves, β and τ are held fixed at 10° and 0.707 , respectively. It will be noticed that the H-plane beamwidth decreases rapidly with increasing ψ and decreasing α . The side-lobe level, shown only for $\alpha = 67.5^\circ$, increases with increasing ψ . Figure 5 shows the variation of the E- and H-plane beamwidths as a function of α with ψ held fixed.

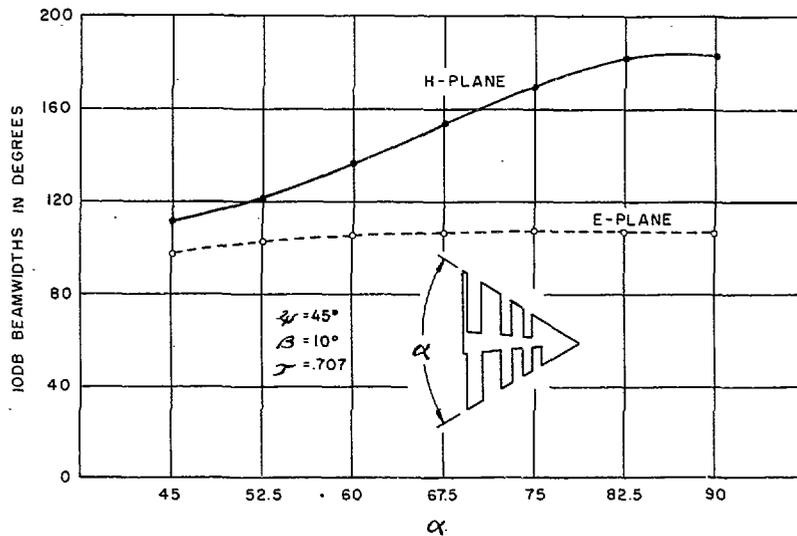


Fig. 5. E and H plane 10 db beamwidths as a function of α .

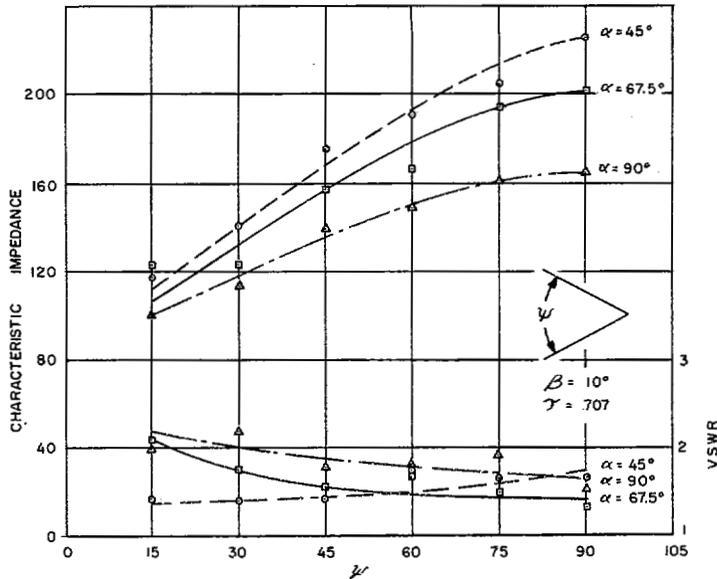


Fig. 6. Characteristic impedance as a function of ψ .

It will be noticed from these figures that the H-plane 10-db beamwidth can be easily controlled over the range of 70° to 200° by the parameters α and ψ . However, it is evident that the E-plane beamwidth is quite insensitive to these parameters since the variation is only from 90° to 110° . Although it has not been investigated thoroughly, some additional control of the E-plane beamwidth can be obtained with the parameters β and τ .

Because of their frequency independence, the pattern characteristics of this log periodic feed are ideally suited for feed applications with H-plane F/D ratios of 0.2 to 0.8 and E-plane F/D ratios of 0.45 to 0.65. For larger F/D ratios a multielement array of log periodic structures may be used.

Impedance Characteristics

If the input impedance of a log periodic antenna is plotted on a Smith Chart over a frequency range of several periods, it will be found that the locus forms a circle with the center lying on the zero reactance line. The characteristic impedance of the antenna is defined as the geometric mean of the maximum and minimum real values on the locus. The vswr referred to this characteristic impedance is then simply equal to the ratio of the maximum impedance to the characteristic impedance.

The variation of the characteristic impedance and vswr with the angles α and ψ is illustrated in figure 6. It will be noticed that the characteristic impedance decreases as ψ is decreased and α is increased. Except for very small values of ψ the vswr is less than 2:1. For the ψ values of most interest, the characteristic impedance ranges from approximately 100 to 200 ohms. Thus, it is necessary to use a wide-band technique to match this impedance to a coaxial or a balanced line. The tapered

line transformers^{7, 8} are ideally suited for matching this impedance over theoretically unlimited bandwidths.

When a matched feed is placed in front of a dish, part of the radiated energy will be reflected from the dish back into the feed. The magnitude of the reflection coefficient due to the dish reflection is given by $\frac{g\lambda}{4\pi F}$ where g is the feed gain along the axis of the dish. For a feed with optimum patterns for a dish with F/D = 0.5, this formula implies that D/λ should be greater than four in order to keep the reflection coefficient less than 1/3 (vswr less than two). Although this mismatch can be compensated for narrow band applications, no wide-band compensation methods are available. Thus, it is usually necessary to live with this additional mismatch for wide-band applications.

In general, free space vswr's ranging from 1.5 to 2.0 may be expected with a log periodic feed. When placed in front of a dish, the vswr will increase somewhat depending upon the focal length.

Phase Center

In general, the phase centers of a log periodic antenna do not lie at the vertex or feed point. Rather, they lie at a fixed electrical distance from the feed point. This distance, as measured in wavelengths, is approximately independent of frequency. This means that if the frequency is changed, the phase center of the feed will move relative to the feed point.

The effect of displacing the phase center from the focal point along the dish axis is a quadratic phase error in the dish aperture distribution. If Δ represents the displacement of the phase center from the focal point, then the phase difference of the aperture distribution between the center of the dish and the edge

of the dish is given approximately by $\frac{\Delta}{\lambda} \left(1 - \cos \frac{\delta}{2}\right)$ where δ is the aperture angle of the dish. The maximum value of this phase difference should be less than approximately $\lambda/8$ which produces a reduction in dish gain⁶ of approximately 0.25 db. For $F/D = 0.5$, this maximum phase difference requires that the maximum displacement of the phase center from the focal point be less than $\lambda/3$. Thus, it is desirable to design the feed such that the phase center lies within $\lambda/3$ of the vertex if extreme bandwidths are to be achieved.

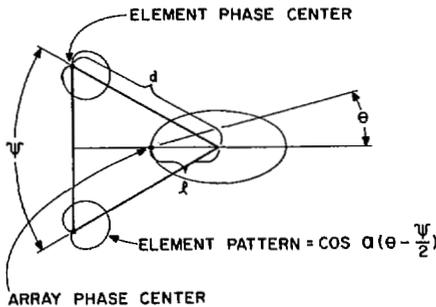


Fig. 7. Coordinate system for the derivation of H plane phase center.

The movement of the phase center with the design parameters may be better understood by considering the following simplified theory. Figure 7 is a schematic diagram of the log periodic feed. The two lines separated by the angle ψ represent the center lines of the individual half structures. A basic characteristic of each of the half structures is that they produce a uni-directional pattern which points in the direction of the element, as sketched in the figure. The distance from the vertex of the half structure to the element E- or H-plane phase center is given by d . Both the E- and H-plane array phase centers will lie on the line bisecting the angle ψ and at a distance ℓ from the vertex. In general, neither the E- or H-plane element or array phase centers will coincide, i.e., d and ℓ will have different values for the E and H planes.

Consider first the array H-plane phase center. The H-plane pattern of the array is given by

$$E = e^{j(\beta \ell - \beta d \cos \frac{\psi}{2}) \cos \theta} \left\{ \cos \alpha \left(\theta + \frac{\psi}{2} \right) e^{j\beta d \sin \frac{\psi}{2} \sin \theta} + \cos \alpha \left(\theta - \frac{\psi}{2} \right) e^{-j\beta d \sin \frac{\psi}{2} \sin \theta} \right\} \quad (1)$$

where $\cos \alpha (\theta \pm \psi/2)$ represents the element pattern. The constant α is determined by

$$\alpha = \frac{\pi}{2\bar{\theta}} \quad (2)$$

where $\bar{\theta}$ is the half-power element beamwidth. A simple investigation of the phase variation for small angles of θ leads to the following formula for the distance of the array phase center from the antenna vertex.

$$\frac{\ell_H}{\lambda} = \frac{d_H}{\lambda} \left[\cos \frac{\psi}{2} - \frac{\pi}{\bar{\theta}} \sin \frac{\psi}{2} \tan \frac{\pi \psi}{4\bar{\theta}} \right] \quad (3)$$

It will be noticed that the array phase center always lies in front of the line joining the two element phase centers by an amount proportional to the second term of equation (3). It also can be seen that the proper choice of $\bar{\theta}$ and ψ will make the H-plane array phase center coincide with the vertex and that for $\psi = 180^\circ$, the phase center will not lie at the vertex as one might originally expect. This derivation neglects the effect of the presence of one half structure upon the current distribution and radiation pattern of the other half structure. Figure 8 is a comparison of the theoretical and measured values of the H-plane phase center as a function of ψ for three values of α . Notice that fairly good agreement is obtained for $\alpha = 45^\circ$ and 90° , but not for $\alpha = 67.5^\circ$.

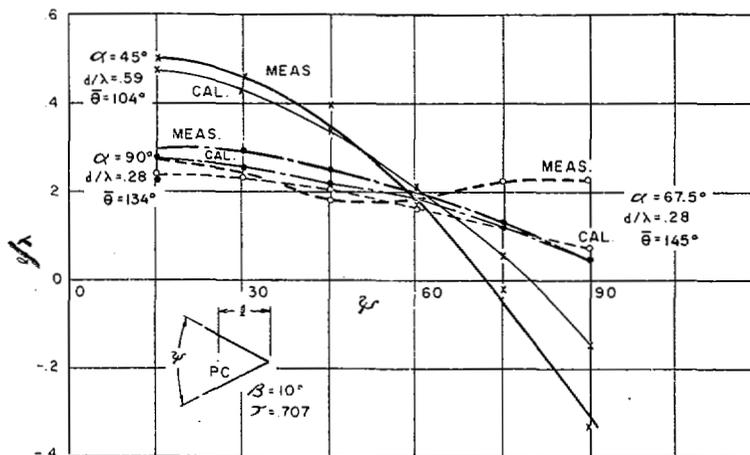


Fig. 8. Comparison of measured and theoretical H plane phase center as a function of ψ .

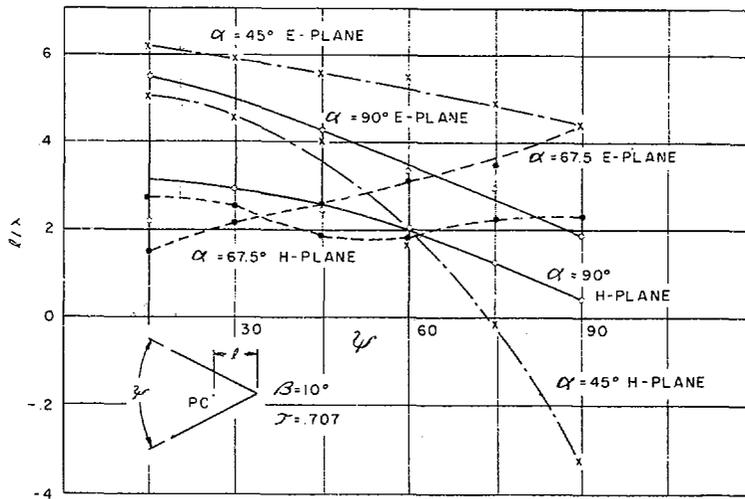


Fig. 9. Variation of E and H plane phase centers with ψ .

Consider next the E-plane array phase center. If the element E- and H-plane phase centers coincide, then the array phase center should lie at a point midway between the two element phase centers, which is at a distance from the vertex given by

$$\frac{l_E}{\lambda} = \frac{d_E}{\lambda} \cos \frac{\psi}{2} \quad (4)$$

If the element E- and H-plane phase centers do not coincide, then the array phase center will lie at some other point, which is a complex function of the element pattern and phase centers. Figure 9 shows the variation of both the E- and H-plane phase centers with the angle ψ for the same values of α as shown in figure 8. Again, the phase centers for $\alpha = 45^\circ$ and 90° move as expected, but for $\alpha = 67.5^\circ$ they move in a very peculiar fashion, especially the E-plane phase center.

The variation of the E- and H-plane phase centers with the angle α for ψ fixed at 45° is shown in figure 10. It will be noticed that the E-plane and H-plane phase centers do not coincide and that the results are again peculiar in that the curves do not decrease monotonically as α is increased.

Figure 11 shows the variation of the E-plane phase center with frequency over a half period. Although all of the structures exhibit some variation of phase center (as measured in wavelengths) with frequency, the results for $\alpha = 90^\circ$ are an extreme case. The information given in the previous figures is the average phase center location over a half period.

It is apparent that the phase center characteristics of log periodic antennas are not ideally suited for wide-band feed applications because the E- and H-plane phase centers do not coincide and the phase centers

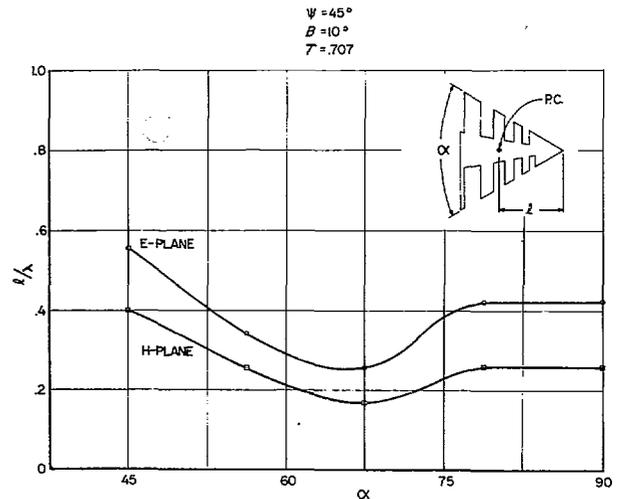


Fig. 10. Variation of E and H plane phase center with α .

move with frequency. However, the phase centers are well-defined, and point source cone half angles of 45° are easily achieved. Moreover, the distance of the phase center from the vertex can be made less than $\lambda/2$ which is satisfactory for most applications.

Feed For a Four-Foot Dish

The investigation of log periodic feeds was directed toward the development of a feed for a four-foot dish with $F/D = 0.5$ to cover the frequency range of 600 to 6000 mc. The final design parameters chosen are $\alpha = 60^\circ$, $\beta = 10^\circ$, $\psi = 45^\circ$, $\tau = 0.707$. The pattern characteristics of this feed are E- and H-plane average

$\psi = 45^\circ$
 $B = 10^\circ$
 $\gamma = .707$
 FREQUENCY RANGE 2000-2380 MC

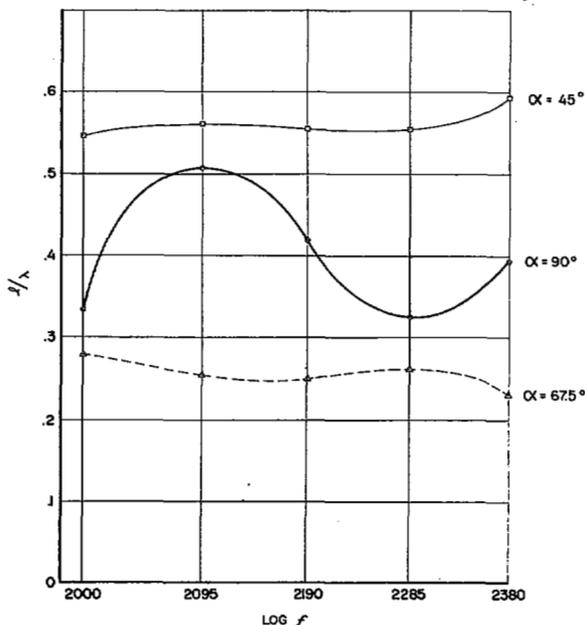
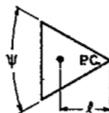


Fig. 11. Variation of E plane phase center with frequency over a half period of frequency.

10-db beamwidths of 105° and 135° respectively, and an average side-lobe level in the order of 15 db. The patterns for this feed are shown in figure 3. It should be pointed out that the patterns for 600 and 5950 mc are near the low and high cutoff frequencies for the feed. Patterns over the major part of this frequency range are similar to those for 2025 mc. The distance to the E- and H-plane phase centers from the vertex are approximately 0.3 and 0.2 of a wavelength respectively. The desired 10-db beamwidth for the dish is about 105° (see figure 2). Thus, the H-plane beamwidths of the feed are much larger than desired. When this feed choice was made, it was thought that it would be better to accept less taper in the H plane in order to obtain a phase center as near the vertex as possible. Measurements have indicated that it probably would have been better to use a feed with $\alpha = 45^\circ$ and accept the larger phase center variation. Figure 12 is a picture of the four-foot dish with the log periodic feed. The tripod arrangement was constructed so that the location of the feed could be adjusted to determine the effects of phase center variation.

The variation of gain with feed placement is shown in figure 13 for six different frequencies over the frequency range of 600 to 6000 mc. The quantity Δ/F represents the relative displacement of the feed along the dish axis with the positive direction being toward the dish. The point 0 corresponds to placing the feed vertex at the focal point. For the various curves, the diameter of the dish in wavelengths is given. For the

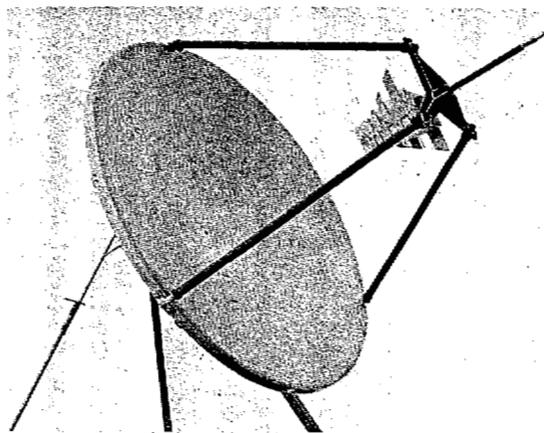


Fig. 12. Four-foot dish with log periodic feed.

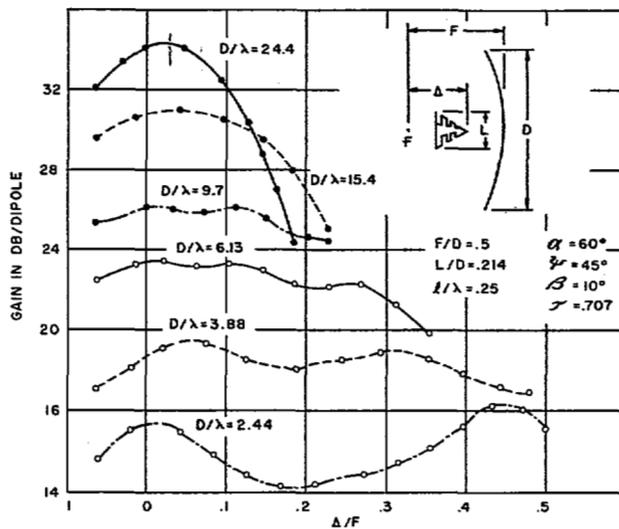


Fig. 13. Reflector gain as a function of feed placement over a 10:1 frequency range.

lower curves, it is noticed that the gain varies in an approximate sinusoidal fashion due to the effect of the feed back lobe on the dish gain. It will be noticed that if Δ/F is set between 0 and 0.025 that the loss in gain due to variation of phase centers is less than 0.5 db over the complete frequency range. The final results to be given on the patterns and impedance are for the case of $\Delta/F = 0.025$.

Secondary Pattern Characteristics

Logarithmic plots of the secondary pattern for the four-foot dish are given in figure 14. Each radial division equals 5 db. At the lowest frequency, where the diameter of the dish is only 2.44 wavelengths, the

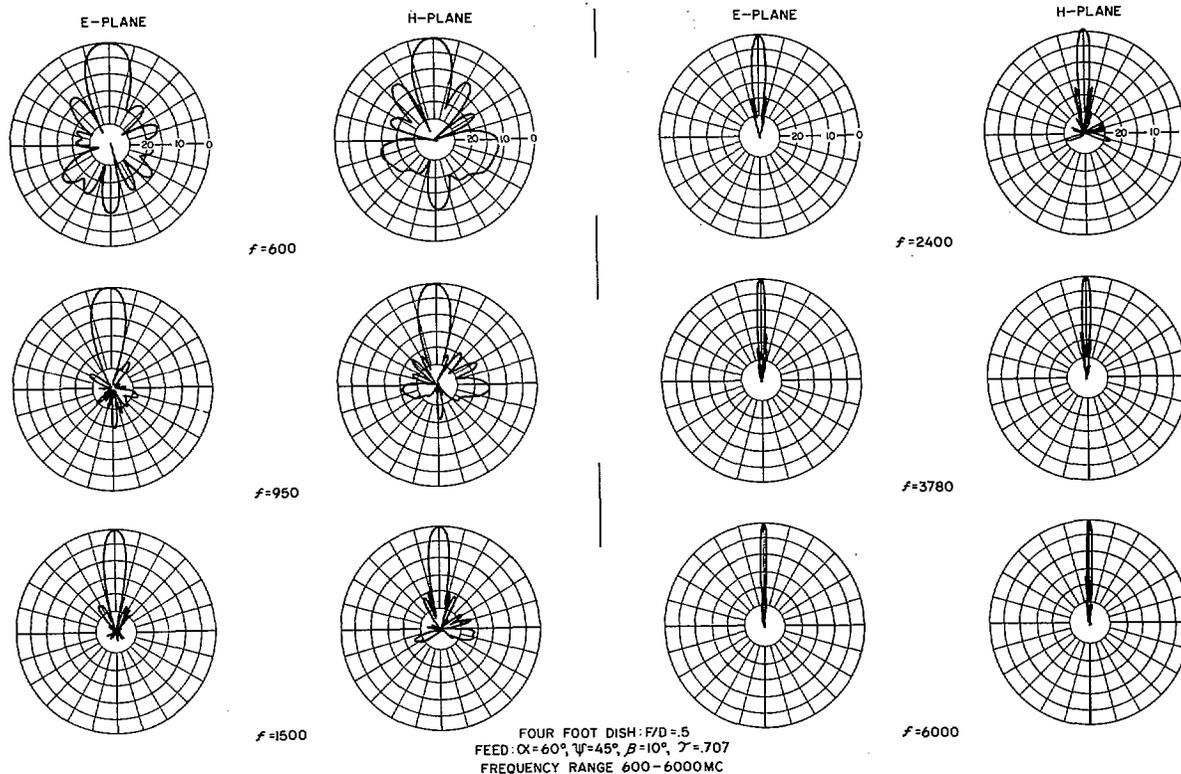


Fig. 14. Patterns over a 10:1 frequency range.

side lobes and back radiation are quite high, as would be expected. Figure 15 summarizes the gain, beamwidths, and side-lobe level for this dish. The product of the dish diameter in wavelengths times the half-power beamwidths in radians has an average value of

about 1.15 for the E plane and 1.09 for the H plane. This may be compared to the case for uniform circular aperture illumination for which this product is theoretically 1.02. The gain factor for the dish is on the order of 0.75.

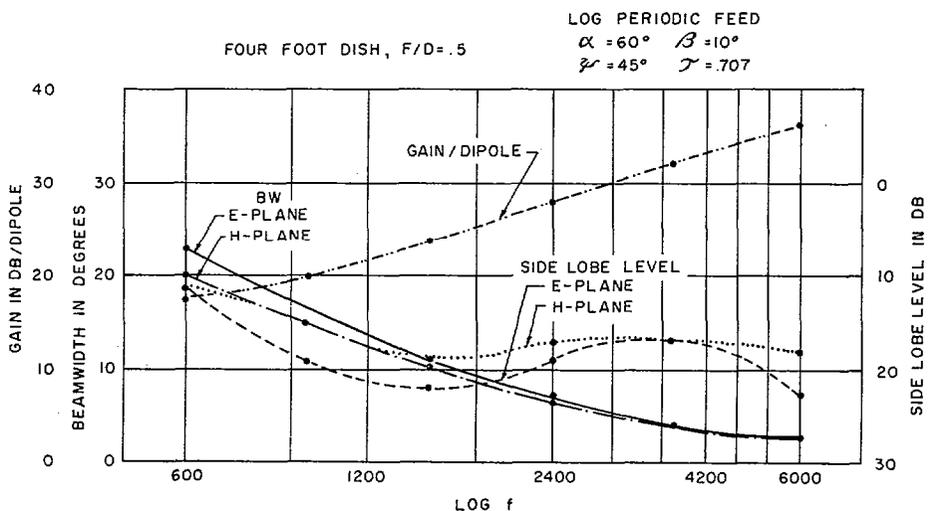


Fig. 15. Gain dipole, beamwidths, and sidelobe level as a function of log f over a 10:1 frequency range.

It will be noticed that the H-plane side-lobe level is higher than the E-plane side-lobe level which is explained by the wider H-plane primary pattern beam-width. For frequencies about 2000 mc it would be expected that the E-plane side-lobe level should be 20 db or less. Possible causes for this discrepancy are aperture blocking, back radiation from the feed, and phase error in the aperture distribution.

Aperture Blocking

For operation over a 10:1 frequency range, the feed is ten times larger in size than is required at the high end of this range. Attempts were made to measure the total absorption and scattering cross section of the feed over this frequency range. The method used is described in reference 9. It was found that the measurements were quite difficult to perform accurately because of the supporting structure required to hold the feed in position. The measurements indicated roughly that the total scattering and absorption cross section was approximately equal to the physical cross section of the feed regardless of the frequency. Since the physical cross section of the feed is approximately 1/25 the area of the dish, the aperture blocking of the feed should not cause more than a 2 or 3 db increase in side-lobe level.

Primary Feed Construction

A close-up view of the sheet trapezoidal-tooth log periodic antenna and the feed cable is shown in figure 16. For an upper frequency limit of 6000 mc, it is quite difficult to bring in a coaxial line of appreciable size along one of the half structures without distorting the radiation patterns and input impedance at the high end of the frequency range. Consequently, the tapered line balun illustrated in figure 17 was used to feed the structure. Briefly, this consists of a coaxial line with the outer conductor gradually opening up in a prescribed theoretical manner and finally tapered to form a balanced line. Baluns of this type have been built to cover frequency ranges of 50:1 with the vswr less than 1.3. The impedance at the balanced end of this tapered line balun was adjusted to be 150 ohms for purposes of matching the antenna over a wide frequency range. Figure 18 shows the vswr at the input of a tapered line balun for the feed in free space and in front of the dish. The vswr is less than 2:1 over a large portion of the 10:1 frequency range and rises to 3:1 at only two points

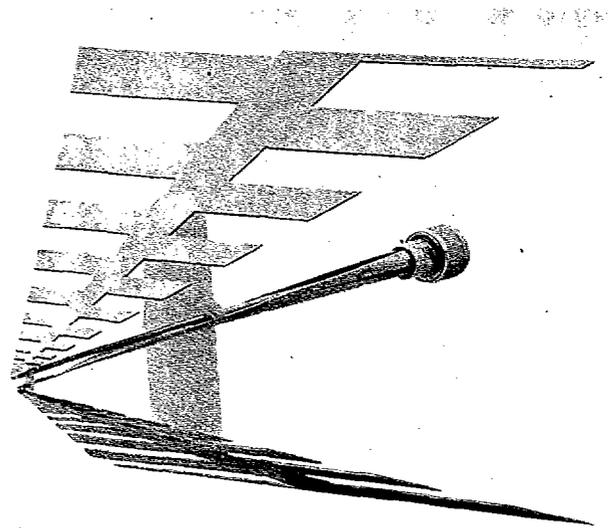


Fig. 16. Log periodic feed with tapered line balun.

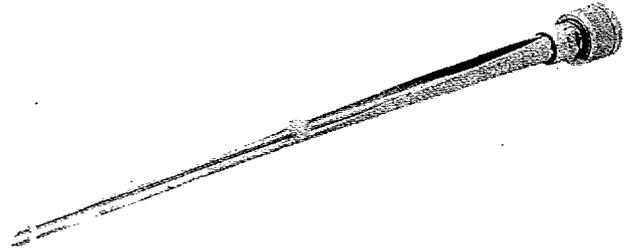


Fig. 17. Tapered line balun.

over this range. The high vswr in the frequency range near 800 mc is due to the reflections from the dish. At this frequency, the dish diameter is less than four wavelengths, which does not satisfy the condition established in a previous section. The vswr could possibly be improved considerably at the high end of this frequency range by taking more care in the design of the tapered line balun and front end of the feed.

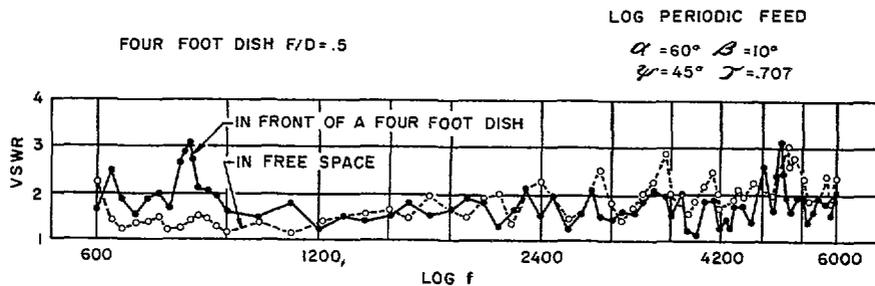


Fig. 18. VSWR as a function of frequency over a 10:1 bandwidth.

Conclusions

Log periodic antennas are well suited as extreme bandwidth feeds for reflectors and lens. The impedance characteristics are satisfactory for applications which do not require v_{swr} 's less than 2:1. The movement of the phase center with frequency produces only a small degradation of the gain and side-lobe level. Independent control of the E- and H-plane beamwidths allows the design of feeds for a variety of reflector and lens shapes. The H-plane beamwidth for a feed consisting of two half structures may be varied over a wide range by changing the parameters α and ψ . Although the E-plane beamwidth is insensitive to these parameters, it may be decreased easily for large F/D ratios by using a multielement array.

Acknowledgement

It is a pleasure to acknowledge the assistance of Forrest G. Arnold in constructing and testing the antenna models.

Bibliography

1. R. H. DuHamel and D. E. Isbell, "Broadband Logarithmically Periodic Antenna Structures", 1957 IRE National Convention Record, Part I, pp. 119-128.
2. D. E. Isbell, "Non-Planar Logarithmically Periodic Antenna Structures", University of Illinois, Antenna Laboratory TR #30, February 20, 1958 Contract AF 33(616)-3220.
3. R. H. DuHamel and F. R. Ore, "Logarithmically Periodic Antenna Designs", 1958 IRE Convention Record, Part I, pp. 139-151.
4. R. H. DuHamel and D. G. Berry, "Logarithmically Periodic Antenna Arrays", 1958 IRE Wescon Convention Record, Part I, pp. 161-174.
5. V. H. Rumsey, "Frequency Independent Antennas", 1957 IRE National Convention Record, Part I, pp. 114-118.
6. S. Silver, "Microwave Antenna Theory and Design", Radiation Laboratory Series #12, McGraw-Hill, 1949.
7. S. B. Cohn, "Optimum Design of Stepped Transmission-Line Transformers", Trans. IRE, V. MTT-3, April 1955, p. 16.
8. R. W. Klopfenstein, "A Transmission Line Taper of Improved Design", Proc. IRE, V. 44, Jan. 1956, p. 31.
9. J. T. Bolljahn & W. S. Lucke, "Some Relationships between Total Scattered Power and the Scattered Field in the Shadow Zone", Trans. IRE, V. AP-4, January, 1956, pp. 69-71.