
EE5321/EE7321

Semiconductor Devices and Circuits

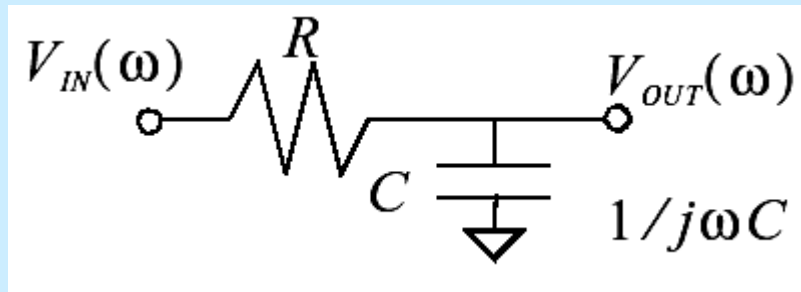
Frequency Response Part1

Impedance network transfer function

- Impedance network transfer function:

$$H(\omega) = \frac{V_{\text{out}}(\omega)}{V_{\text{in}}(\omega)}$$

where $H(\omega)$, $V_{\text{out}}(\omega)$ and $V_{\text{in}}(\omega)$ are phasors



$$H(\omega) = \frac{V_{\text{out}}(\omega)}{V_{\text{in}}(\omega)} = \frac{\frac{1}{j\omega C}}{R + \left(\frac{1}{j\omega C}\right)} = \frac{1}{1 + j\omega R C}$$

$H(\omega)$ in polar coordinates

- $H(\omega)$ is represented by its amplitude and phase
- Amplitude $|H(\omega)|$

$$|H(\omega)| = \sqrt{H(\omega) \bullet H^*(\omega)}$$

- Phase $\angle \theta$

$$\theta(\omega) = \arctan \left\{ \frac{\text{Im}[H(\omega)]}{\text{Re}[H(\omega)]} \right\}$$

- If $H(\omega) = N(\omega) / D(\omega)$ then:

$$\text{Re}[H(\omega)] = \text{Re}[N(\omega) \cdot D^*(\omega)]$$

$$\text{Im}[H(\omega)] = \text{Im}[N(\omega) \cdot D^*(\omega)]$$

$H(\omega)$ for the RC circuit

- Amplitude

$$\sqrt{H(\omega) \bullet H^*(\omega)} = \sqrt{\left(\frac{1}{1 + j \omega R C} \right) \bullet \left(\frac{1}{1 - j \omega R C} \right)}$$

$$\sqrt{H(\omega) \bullet H^*(\omega)} = \sqrt{\left(\frac{1}{1 + (\omega R C)^2} \right)}$$

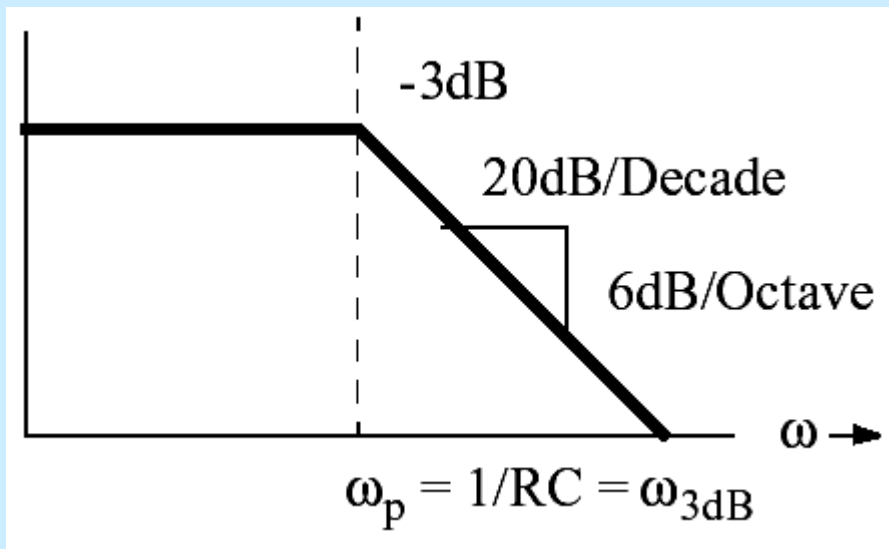
- Amplitude in Decibels $|H(\omega)|_{\text{dB}} = 20 \bullet \log[|H(\omega)|]$
- For a -3dB reduction on the magnitude

$$-3 = 20 \bullet \log[|H(\omega_{3dB})|] \quad \rightarrow \quad |H(\omega_{3dB})| = 0.7079$$

Bode Plot RC circuit – Amplitude

$$|H(\omega_{3dB})| = \sqrt{\left(\frac{1}{1 + (\omega_{3dB} RC)^2}\right)} = 0.7079 \quad \Rightarrow \quad \omega_{3dB} = \omega_p = \frac{1}{RC}$$

$$|H(\omega)| = \sqrt{\frac{1}{1 + \left(\frac{\omega}{\omega_{3dB}}\right)^2}} \quad \text{For } \omega \gg \omega_{3dB} \quad |H(\omega)| \approx \sqrt{\frac{1}{\left(\frac{\omega}{\omega_{3dB}}\right)^2}}$$



$$|H(\omega)| \approx \frac{\omega_{3dB}}{\omega}$$

Amp drops by 2 when f doubles

Amp drops by 10 every decade

Bode Plot RC circuit - Phase

- $H(\omega)$ phase $\theta(\omega) = \arctan \left\{ \frac{\text{Im}[H(\omega)]}{\text{Re}[H(\omega)]} \right\}$

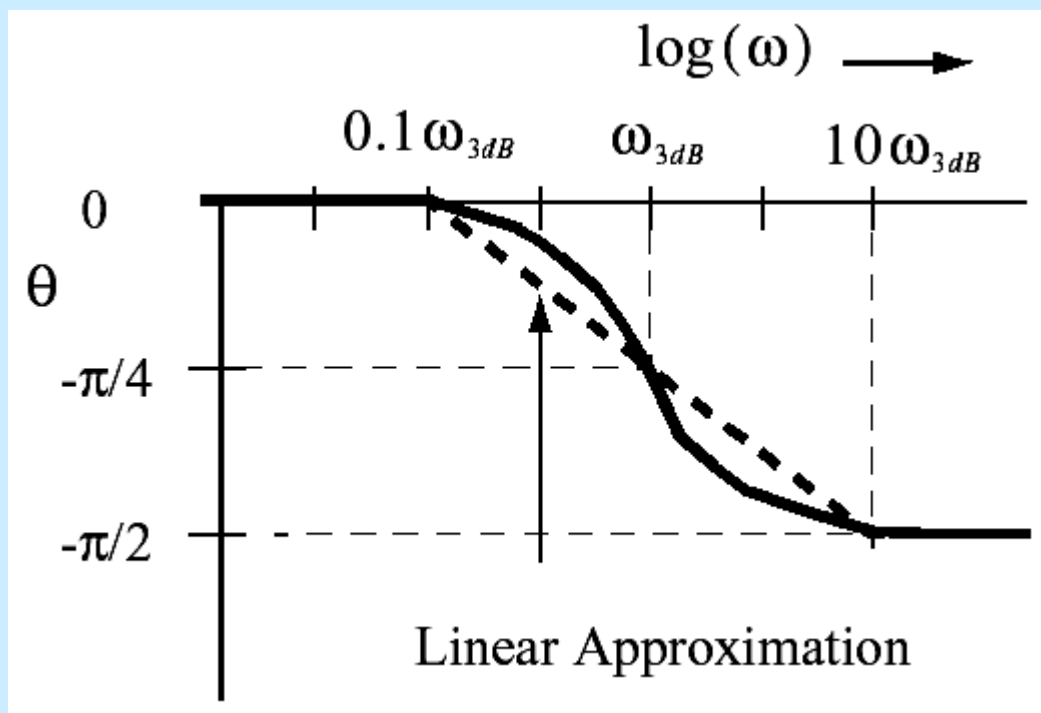
$$H(\omega) = \frac{1}{1 + j \frac{\omega}{\omega_{3dB}}} = \frac{1}{1 + j \frac{\omega}{\omega_{3dB}}} \bullet \frac{1 - j \frac{\omega}{\omega_{3dB}}}{1 - j \frac{\omega}{\omega_{3dB}}} = \frac{1 - j \frac{\omega}{\omega_{3dB}}}{1 + \left(\frac{\omega}{\omega_{3dB}} \right)^2}$$

$$\text{Re}\{H(\omega)\} = \frac{1}{1 + \left(\frac{\omega}{\omega_{3dB}} \right)^2} \quad \text{Im}\{H(\omega)\} = \frac{-\left(\frac{\omega}{\omega_{3dB}} \right)}{1 + \left(\frac{\omega}{\omega_{3dB}} \right)^2}$$

Bode Plot RC circuit - Phase

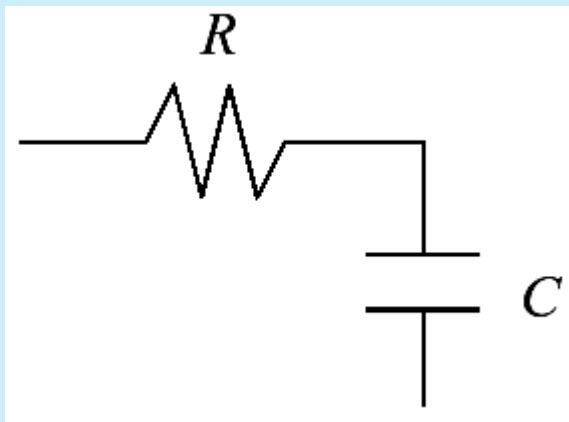
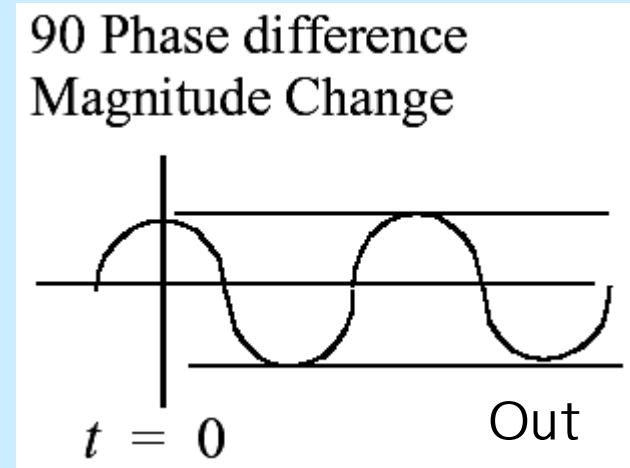
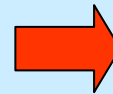
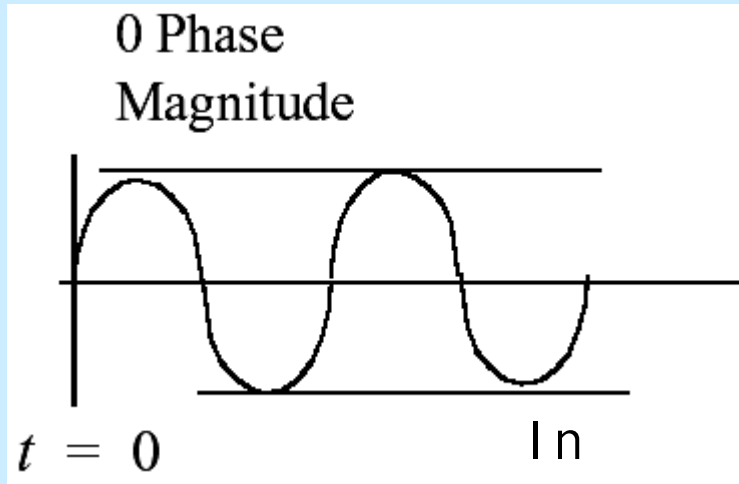
- And the Phase is given by:

$$\theta(\omega) = \arctan\left\{\frac{\text{Im}[H(\omega)]}{\text{Re}[H(\omega)]}\right\} = \arctan\left\{-\frac{\omega}{\omega_{3dB}}\right\} = -\arctan\left\{\frac{\omega}{\omega_{3dB}}\right\}$$



RC circuit - sine wave

- The output wave has amplitude and phase altered by the circuit

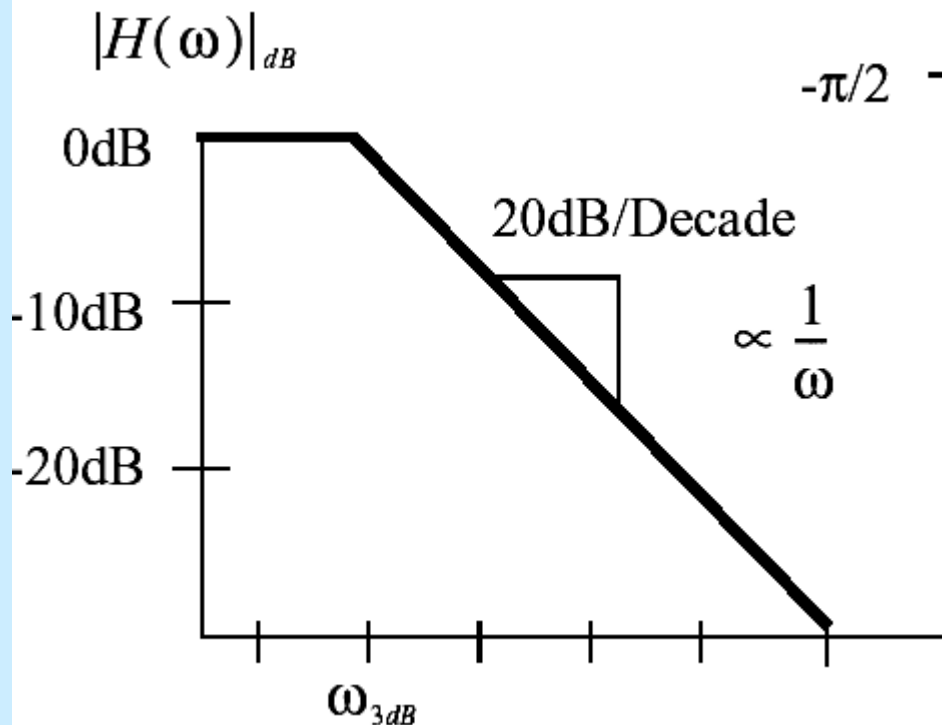
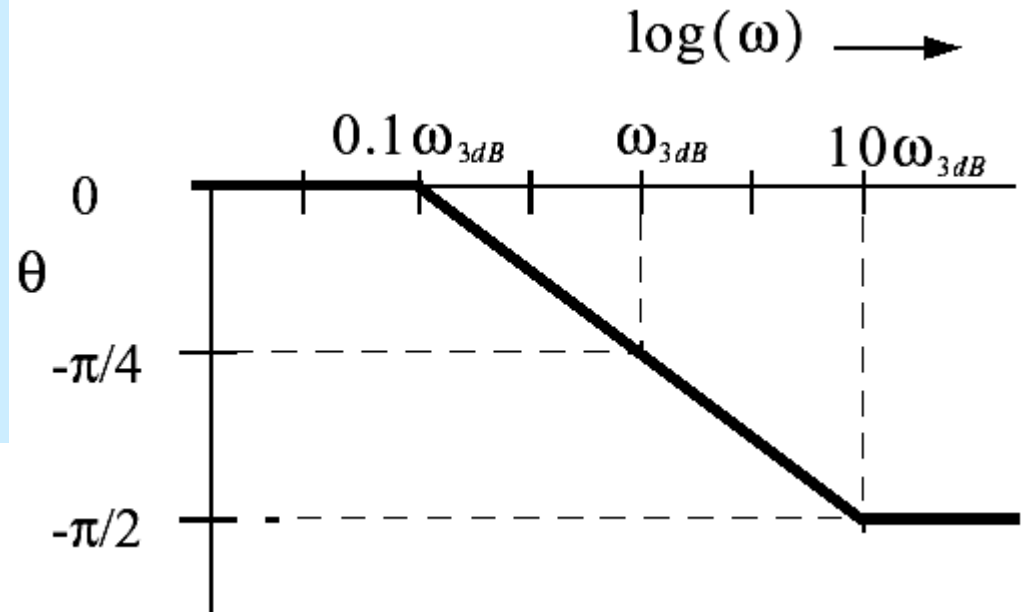


$$H(\omega) = \frac{1}{1 + j \omega R C}$$

Bode Plots – 1 pole

- RC circuit

$$\omega_{3dB} = \omega_p = \frac{1}{RC}$$

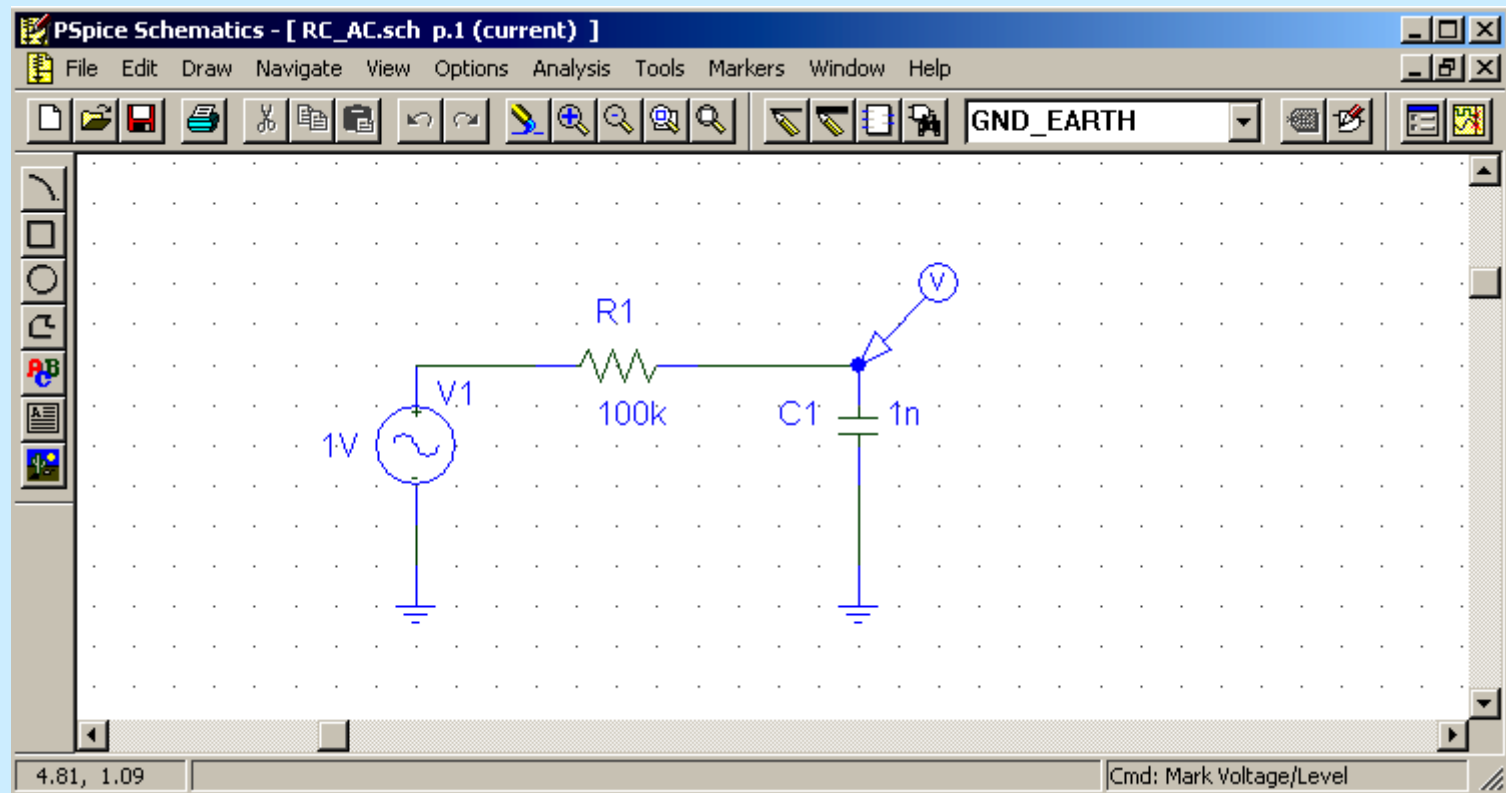


$$\theta(\omega) = -\arctan\left\{\frac{\omega}{\omega_{3dB}}\right\}$$

$$|H(\omega)| \approx \frac{\omega_{3dB}}{\omega}$$

SPICE SIM - RC circuit

- Run AC Sweep with 1V amplitude and freq: 10Hz to 100MHz
- Output DB[V2(C1)/V1(V1)] and P[V2(C1)]



SPIKE SIM - RC circuit

- $\omega_p = 1/RC = 10k \rightarrow f_p = 1.6kHz$

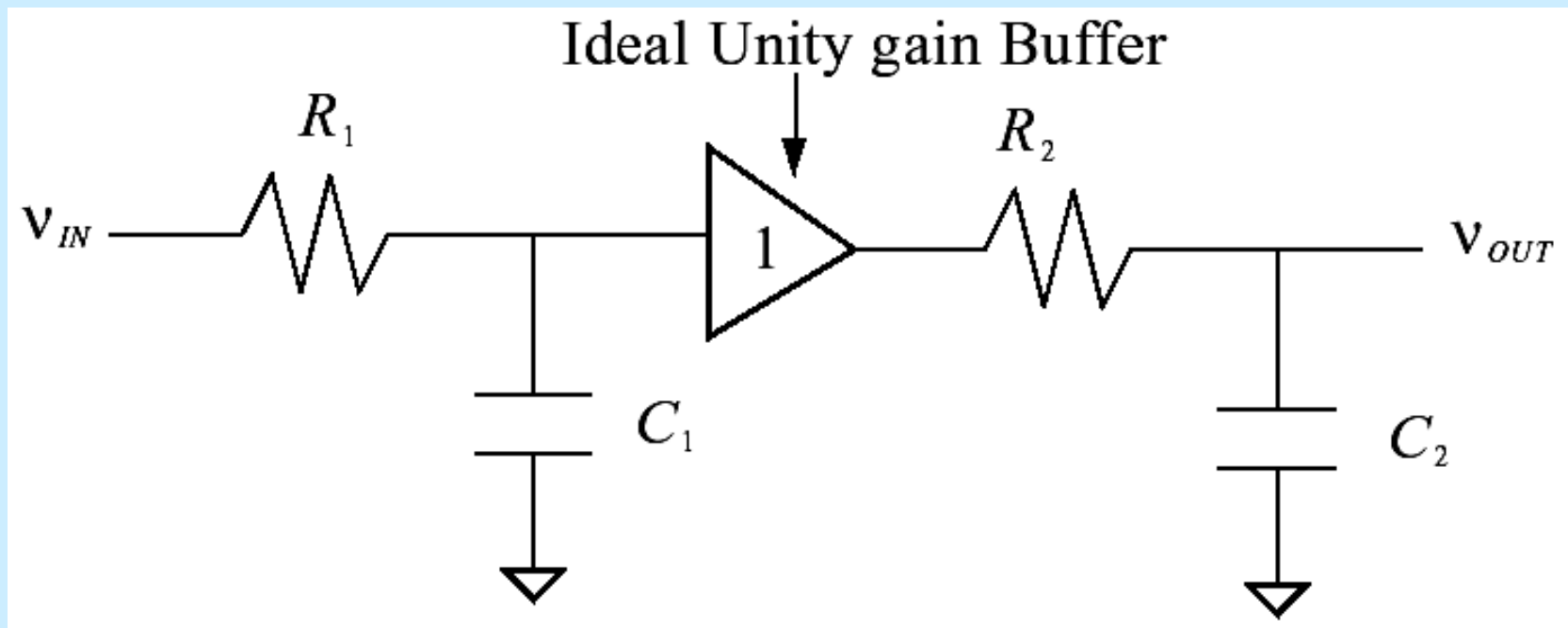


RC circuits in series – 2 poles

- The combination of two RC circuits in series is going to result in 2 poles

$$H(\omega) = \frac{1}{1 + j \frac{\omega}{\omega_{p1}}} \bullet \frac{1}{1 + j \frac{\omega}{\omega_{p2}}}$$

where: $\omega_{p1} = 1/R_1C_1$ and
 $\omega_{p2} = 1/R_2C_2$



RC circuits in series – 2 poles

- Overall transfer function $H(\omega) = H_{\omega_{p1}}(\omega) \bullet H_{\omega_{p2}}(\omega)$

$$H(\omega) = \left[|H_{\omega_{p1}}| \bullet \exp(j \theta_{\omega_{p1}}) \right] \bullet \left[|H_{\omega_{p2}}| \bullet \exp(j \theta_{\omega_{p2}}) \right]$$

$$H(\omega) = |H_{\omega_{p1}}| \bullet |H_{\omega_{p2}}| \bullet \exp(j [\theta_{\omega_{p1}} + \theta_{\omega_{p2}}])$$

$$H(\omega) = |H(\omega)| \bullet \exp(j \theta(\omega))$$

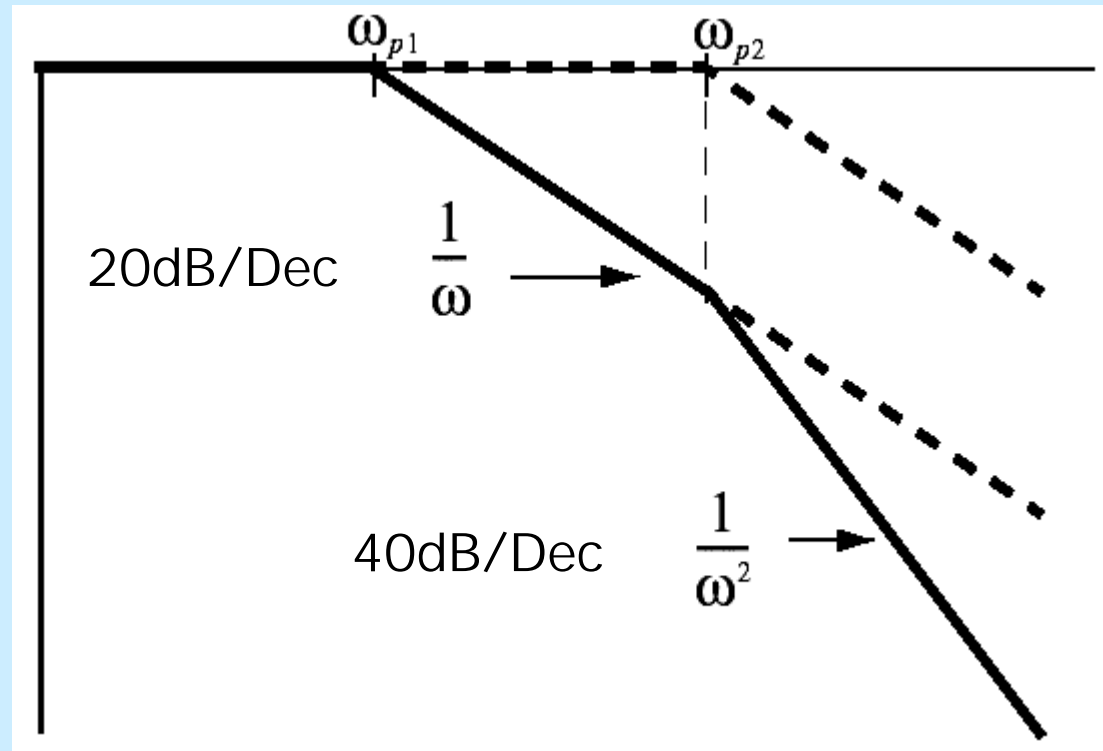
$$\therefore |H(\omega)| = |H_{\omega_{p1}}| \bullet |H_{\omega_{p2}}| \text{ and } \theta(\omega) = \theta_{\omega_{p1}} + \theta_{\omega_{p2}}$$

Amplitude Bode Plot – 2 poles

- Second pole “accelerates” the amplitude reduction

$$20 \bullet \log\{ |H(\omega)| \} = 20 \bullet \log\{ |H(\omega_{p1})| \bullet |H(\omega_{p2})| \}$$

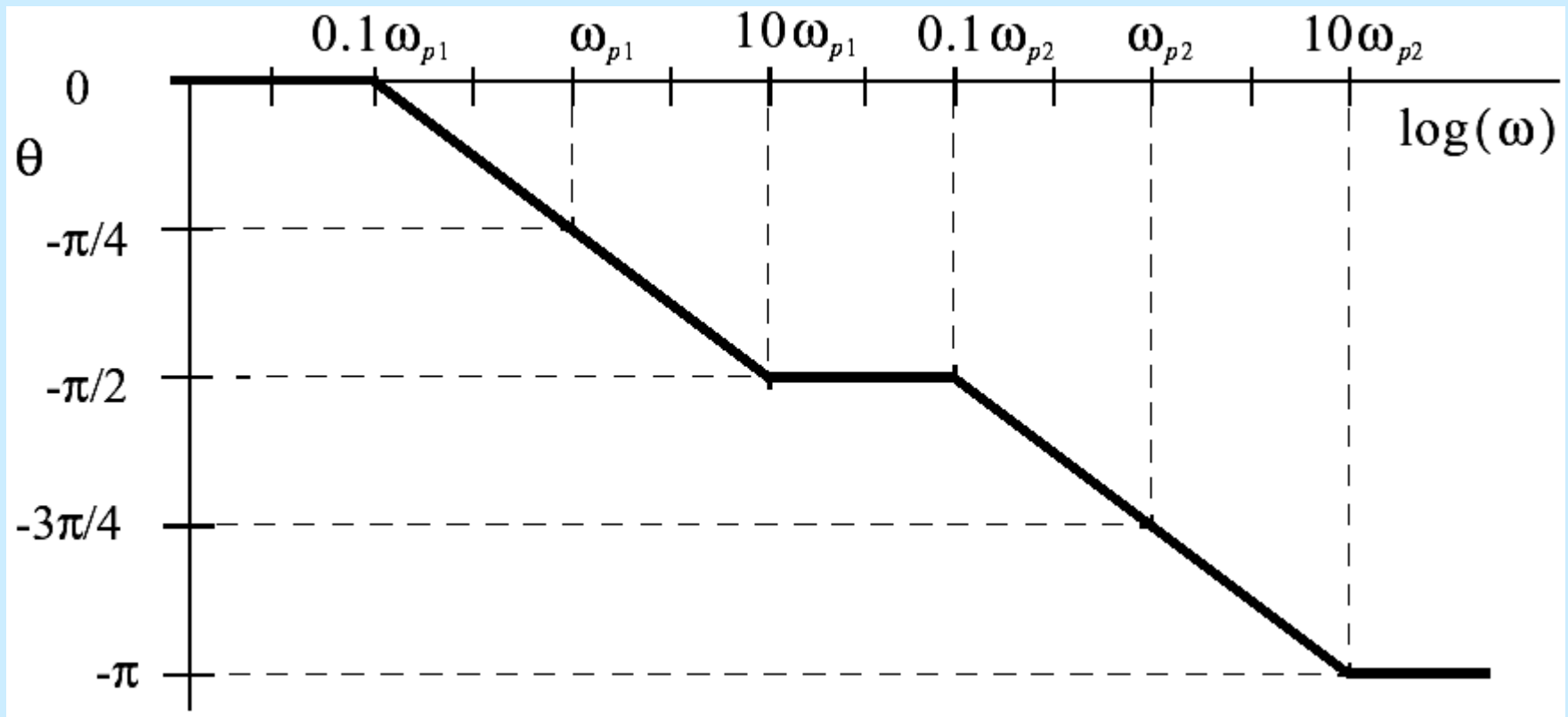
$$20 \bullet \log\{ |H(\omega)| \} = 20 \bullet \log\{ |H(\omega_{p1})| \} + 20 \bullet \log\{ |H(\omega_{p2})| \}$$



Phase Bode Plot - 2 poles

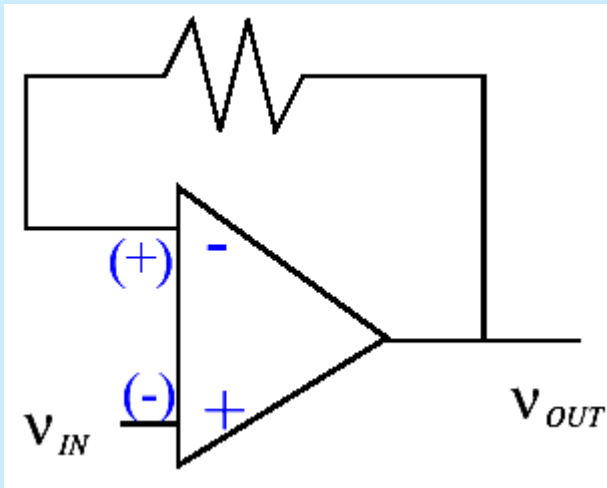
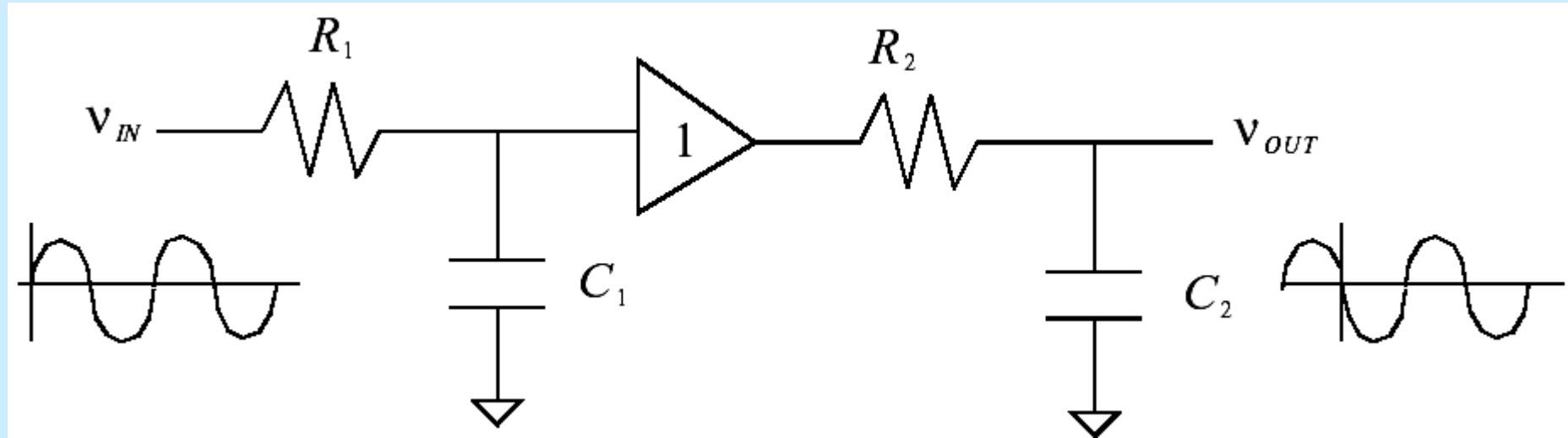
- Second pole adds to the phase shift

$$\theta(\omega) = \theta_{\omega p1} + \theta_{\omega p2}$$



2 poles circuit – 180° phase shift

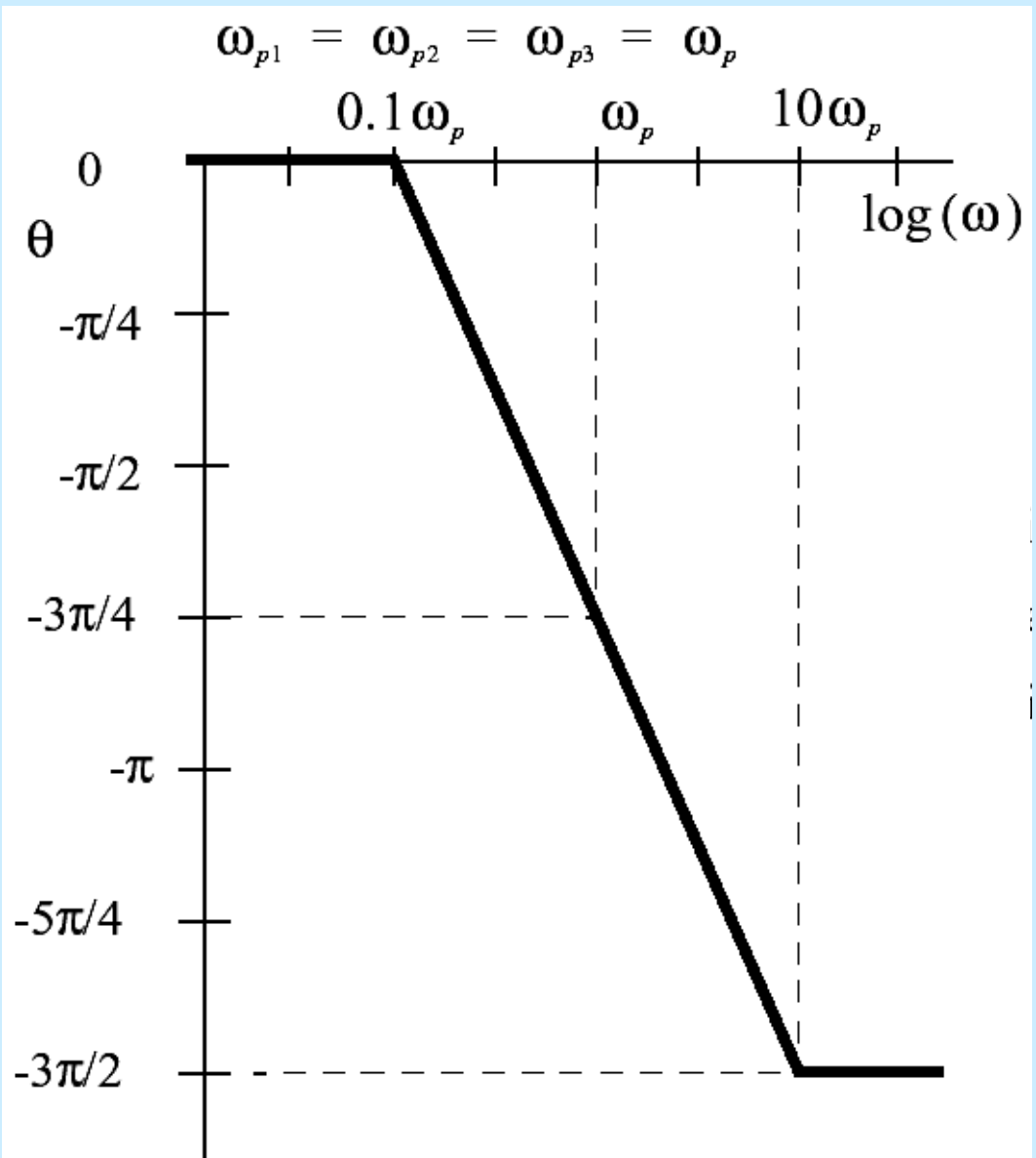
- A phase shift of 180° can be a problem



- If in a feedback loop, a 180° phase shift will turn a negative feedback into a positive feedback
- This results in an unstable system if the loop gain is > 1

Bode Plots – 3 Superimposed Poles

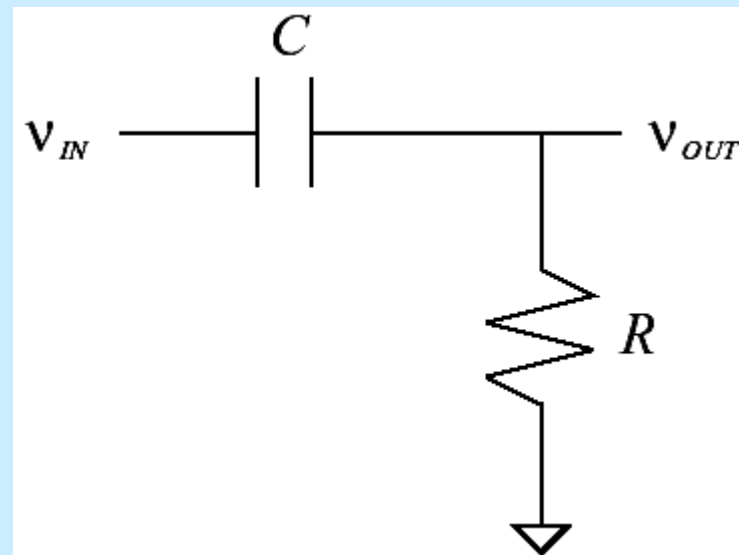
- The phase shift is quite “fast” and “strong”
- When used in a feedback loop will probably result in an unstable circuit



C R circuit - $H(\omega)$

- Circuit has:
 - 1 Zero at $\omega = 0$
 - 1 Pole at $\omega = 1/R C$

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)}$$



$$H(\omega) = \frac{R}{R + \left(\frac{1}{j\omega C} \right)} = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega RC}{1 + j\frac{\omega}{1/RC}}$$

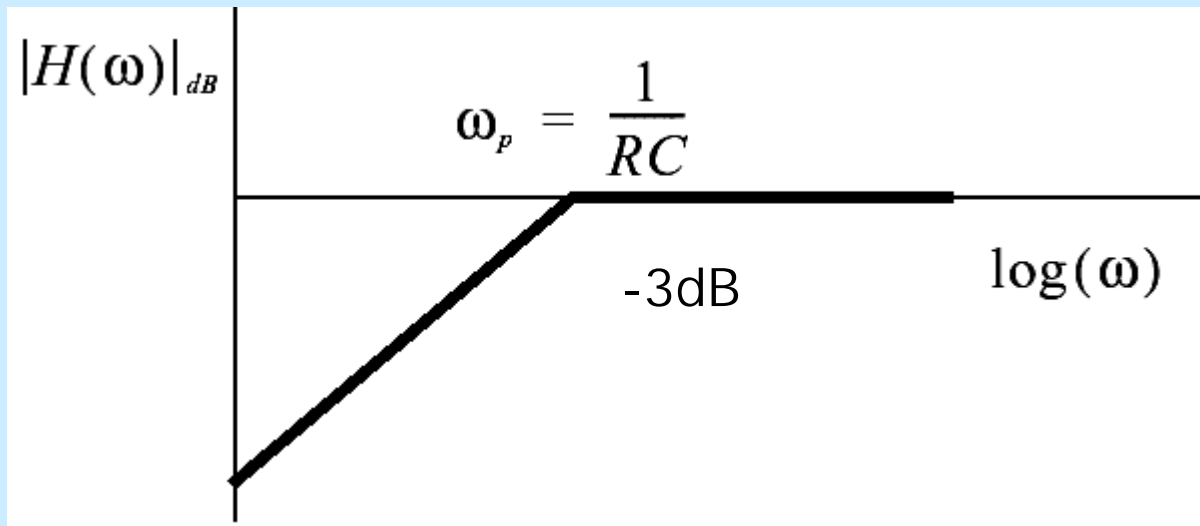
$$|H(\omega)| = \omega RC \bullet \sqrt{\frac{1}{1 + (\omega RC)^2}}$$

C R circuit – Bode plot amplitude

- At $\omega = 0 \rightarrow |H(\omega)| = 0$ and since $|H(\omega)|_{\text{dB}} = 20 \bullet \log[|H(\omega)|]$

$$|H(\omega)|_{\text{dB}} = 20 \bullet \log \left(\omega R C \bullet \sqrt{\frac{1}{1 + (\omega R C)^2}} \right) \rightarrow -\infty$$

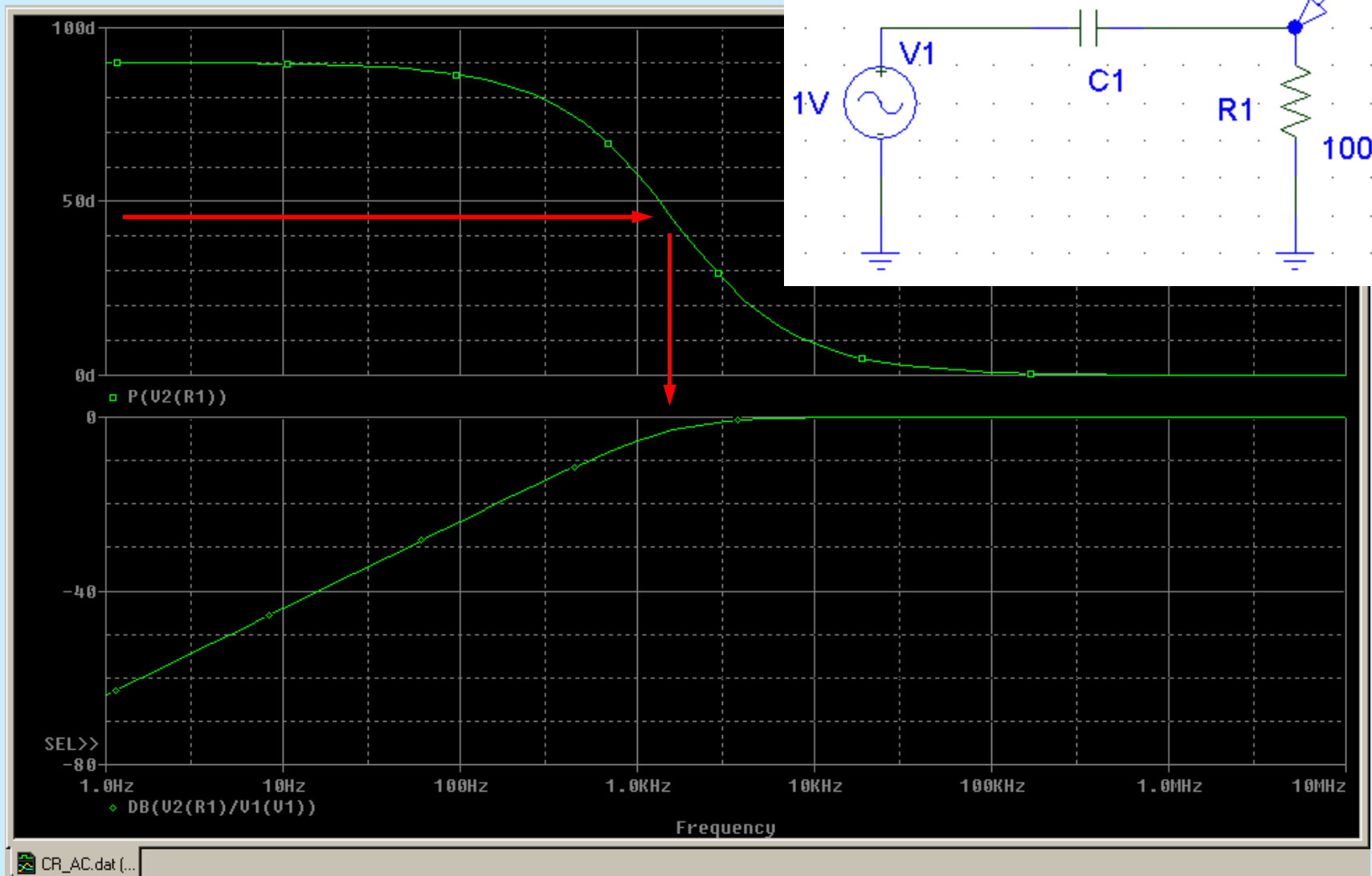
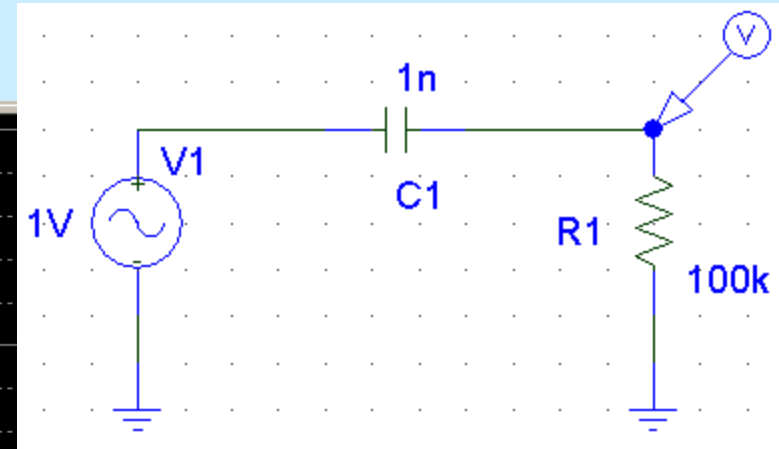
$$|H(\omega_p)|_{\text{dB}} = 20 \bullet \log \left(1 \bullet \sqrt{\frac{1}{1 + (1)^2}} \right) = -3\text{dB}$$



$$\begin{aligned} |H(\omega \gg \omega_p)|_{\text{dB}} &= 20 \bullet \log(1) \\ &= 0 \end{aligned}$$

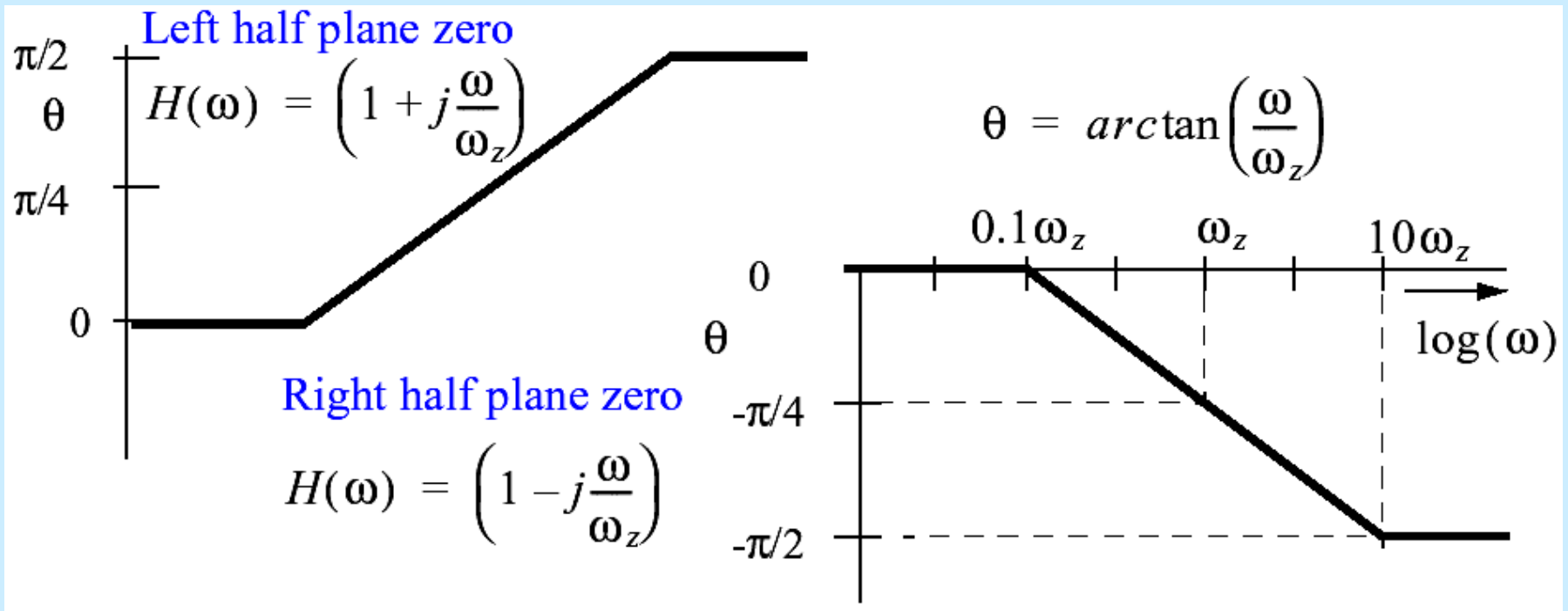
SPIKE SIM - C R circuit

- $\omega_p = 1/RC = 10k \rightarrow f_p = 1.6kHz$



Zero's phase response

- The phase response of a Zero depends on which half plane the Zero is located



$$H(s) = 1 - \frac{s}{s_z}$$

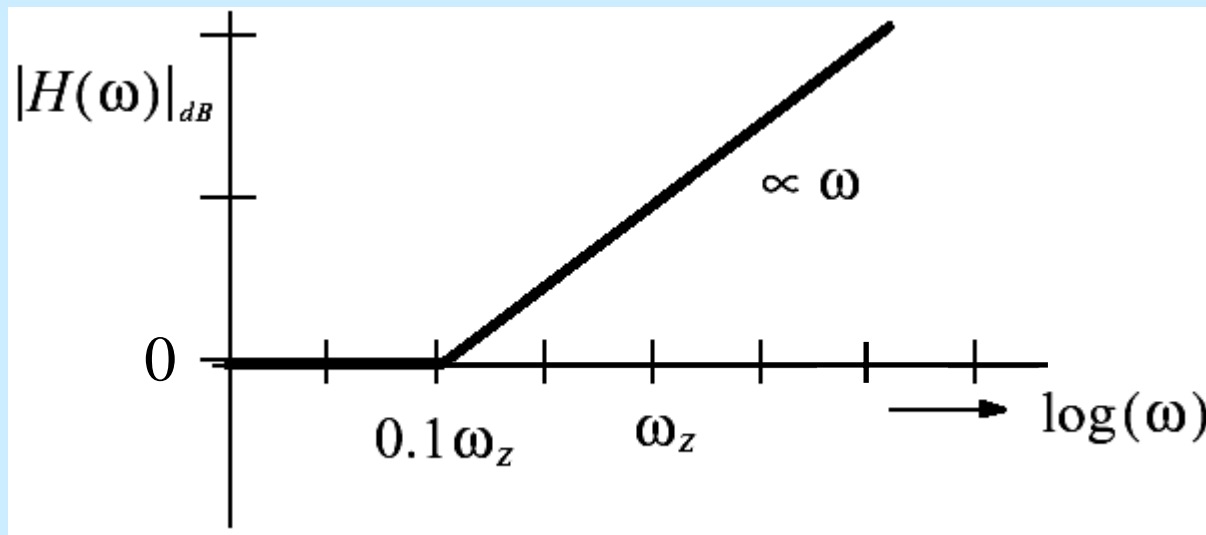
$$s_z = -j \omega_z$$

$$H(s) = 1 + \frac{s}{s_z}$$

Zero's gain response

- For Zero in either half plane the amplitude response is the same

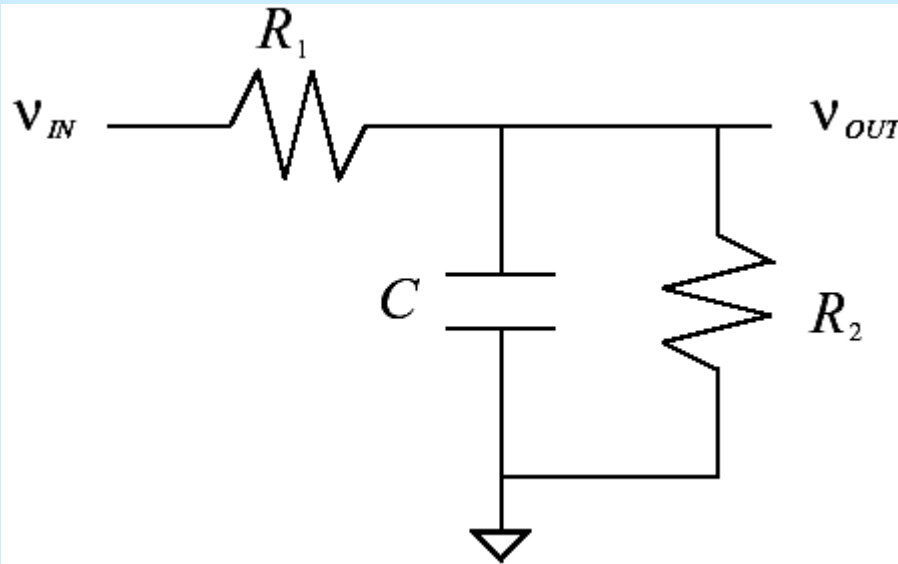
$$|H(\omega)| = \sqrt{1 + \left(\frac{\omega}{\omega_z}\right)^2} \quad \rightarrow \quad |H(\omega)|_{dB} = 10 \bullet \log \left(1 + \left(\frac{\omega}{\omega_z}\right)^2 \right)$$



$$|H(\omega \gg \omega_z)|_{dB} \approx 20 \bullet \log \left(\frac{\omega}{\omega_z} \right)$$

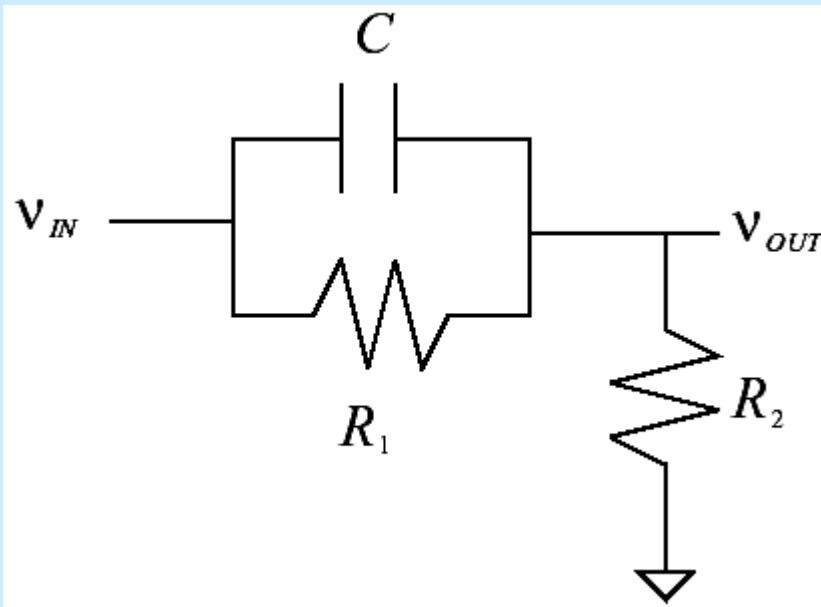
20dB/dec

Transfer function – Other circuits



- 1 Pole

$$H(\omega) = \frac{R_2}{R_1 + R_2} \cdot \frac{1}{1 + j\omega(R_1 || R_2)C}$$



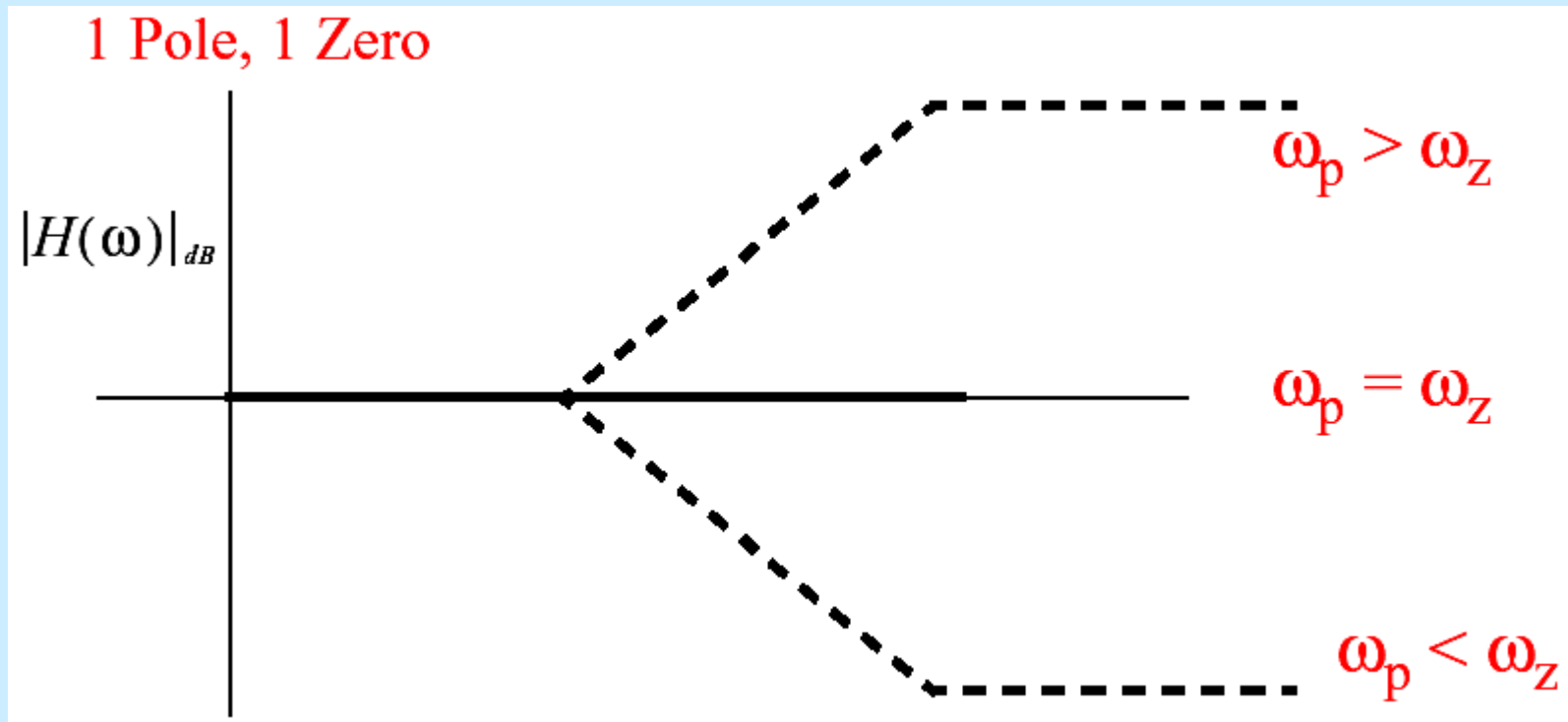
- 1 Pole, 1 Zero

$$H(\omega) = \frac{R_2}{R_1 + R_2} \cdot \frac{1 + j\omega R_1 C}{1 + j\omega(R_1 || R_2)C}$$

1 Pole, 1 Zero response

- The response depends on the relative location of the Pole and the Zero

$$H(\omega) = \frac{1 + j \frac{\omega}{\omega_z}}{1 + j \frac{\omega}{\omega_p}}$$



MOSFET capacitances - circuit

- Specs: t_{ox} (C_{ox}), C_{GSO} , C_{GDO} , C_{GBO} , C_J , PB (ϕ_B)

- Typical Values

$$C_{ox} = 10^{-4} \text{ F/m}^2$$

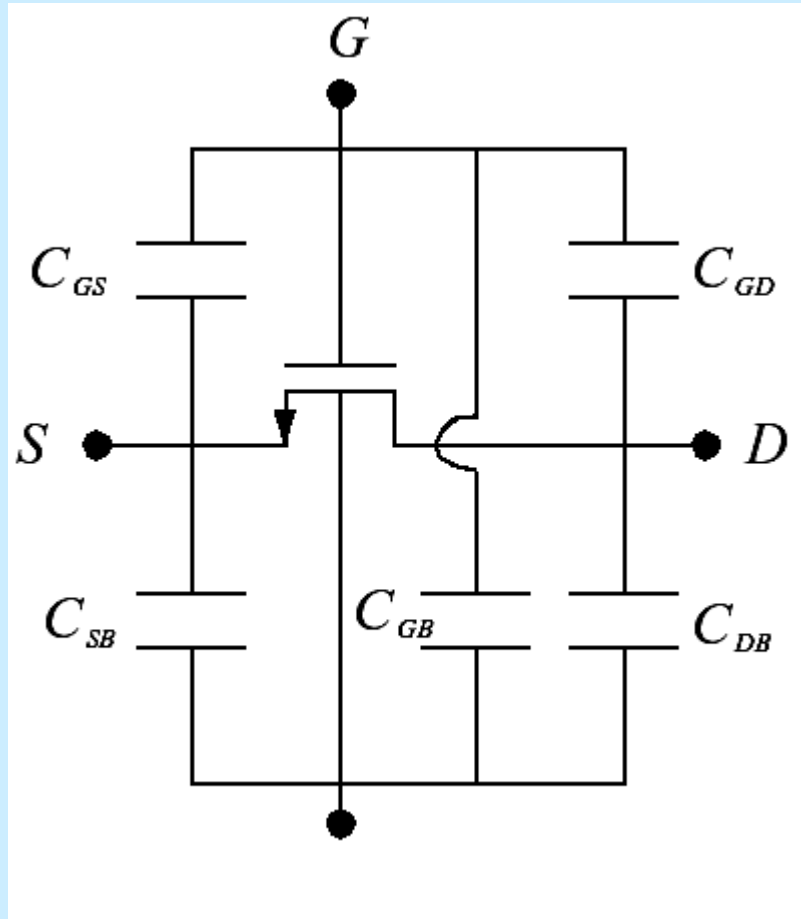
$$C_{GSO} = 5 \times 10^{-10} \text{ F/m}$$

$$C_{GDO} = 5 \times 10^{-10} \text{ F/m}$$

$$C_{GBO} = 4 \times 10^{-10} \text{ F/m}$$

$$C_J = 10^{-4} \text{ F/m}^2$$

$$PB = 0.8 \text{ V}$$



MOSFET capacitances - equations

Saturation

$$C_{GS} = \frac{2}{3} \cdot C_{ox} \cdot L \cdot W + CGSO \cdot W$$

$$C_{GD} = CGDO \cdot W$$

Linear

$$C_{GS} = \frac{C_{ox} \cdot L \cdot W}{2} + CGSO \cdot W$$

$$C_{GD} = \frac{C_{ox} \cdot L \cdot W}{2} + CGDO \cdot W$$

with: PS = Perimeter of Source, AS = Area of Source

MJ = ½ (default), MJSW = 3 (default)

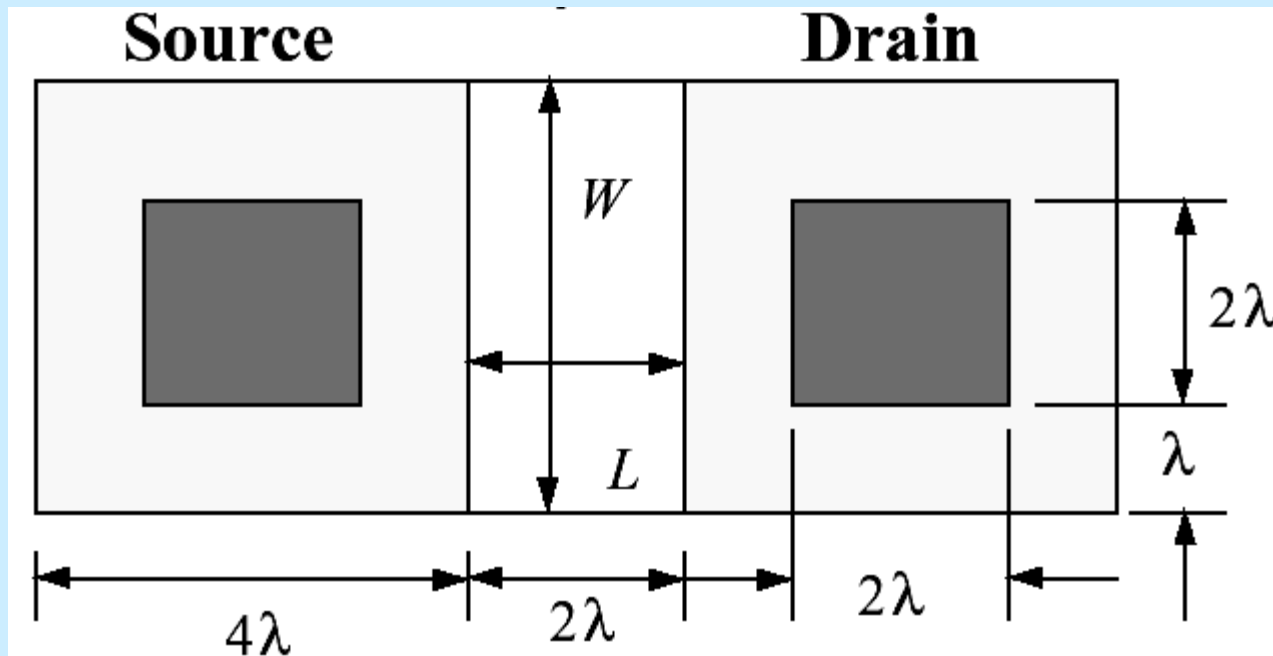
$$C_{SB} = \frac{CJ \cdot AS}{\left(1 + \frac{V_{BS}}{PB}\right)^{MJ}} + \frac{CJSW \cdot PS}{\left(1 + \frac{V_{BS}}{PB}\right)^{MJSW}}$$

a similar equation is used to calculate C_{DB}

$$C_{GB} = CGBO \cdot L$$

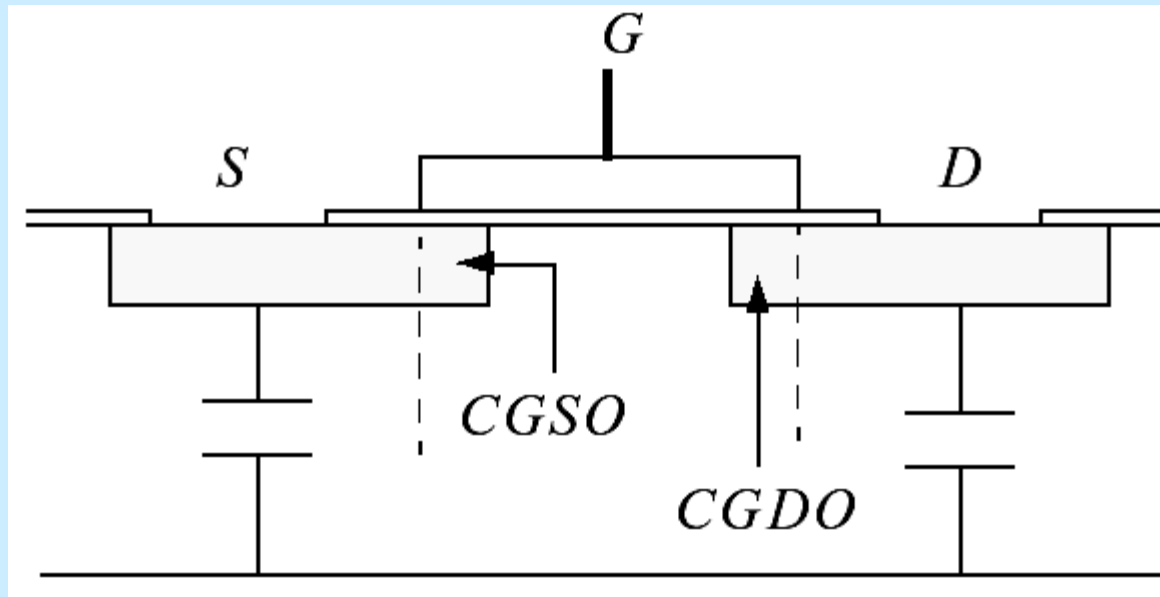
MOSFET - classic layout

- Area of Source = $A_S = 4\lambda \cdot W$
- Area of Drain = $A_D = A_S = 4\lambda \cdot W$
- Perimeter of Source = $P_S = 8\lambda + W$
- Perimeter of Drain = $P_D = 8\lambda + W$



MOSFET - SPICE attributes

M1 1 2 3 4 NMOS L=2U W=2U
+ AS=4p AD=4p PS=6U PD=6U



- Overlap capacitances are calculated using W
- Capacitance to body have area and perimeter terms

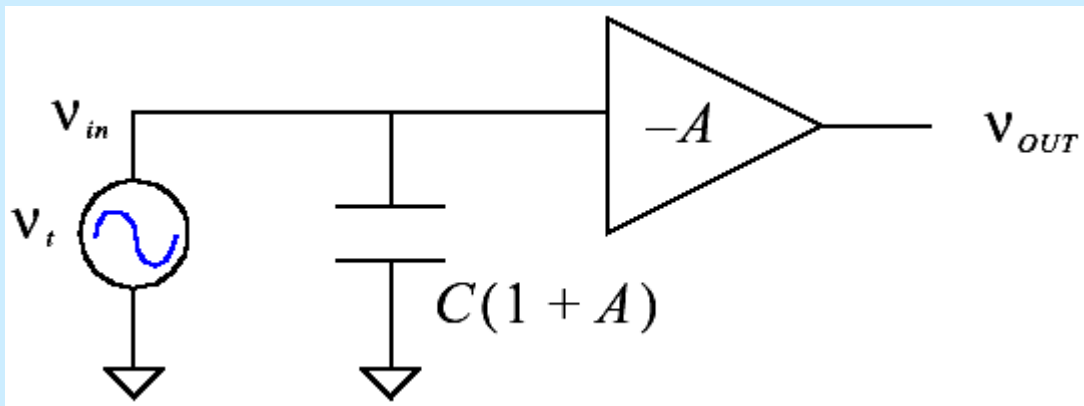
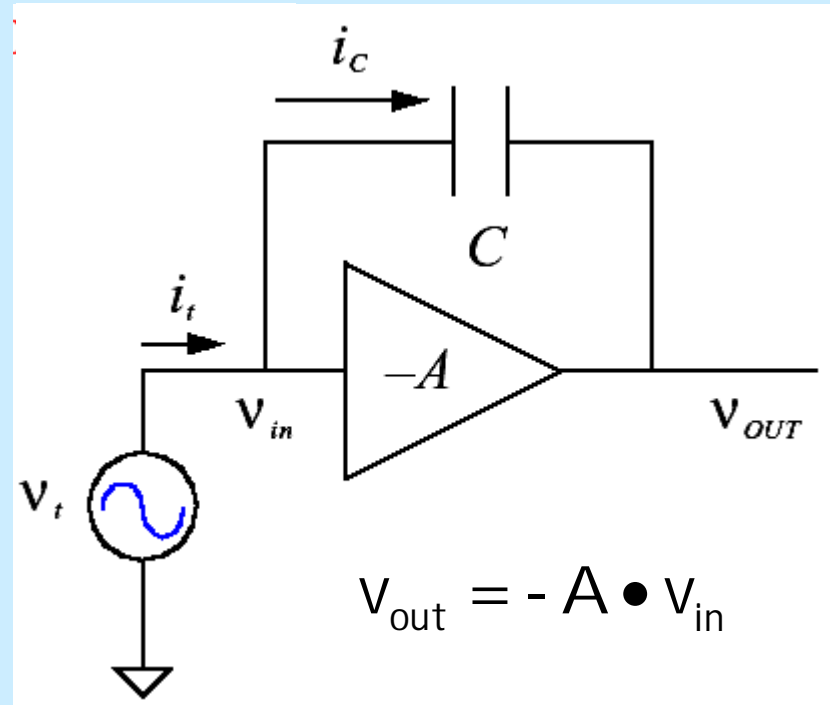
Miller approximation

- Capacitance between input and output appears multiplied by the gain at the input

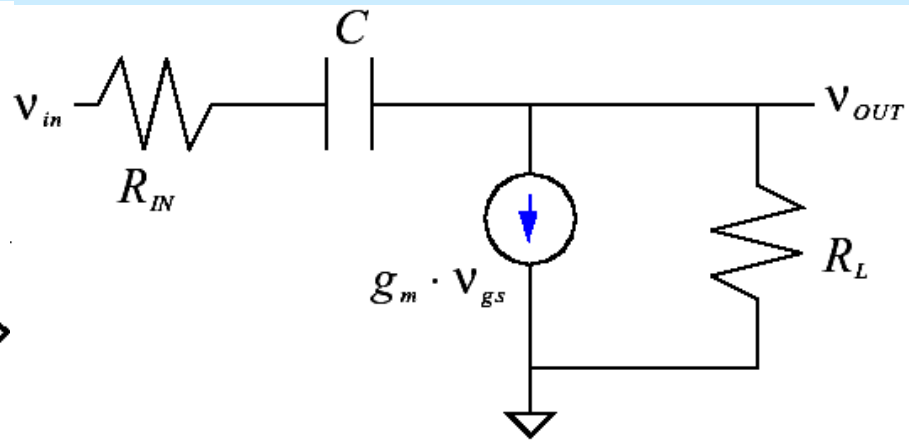
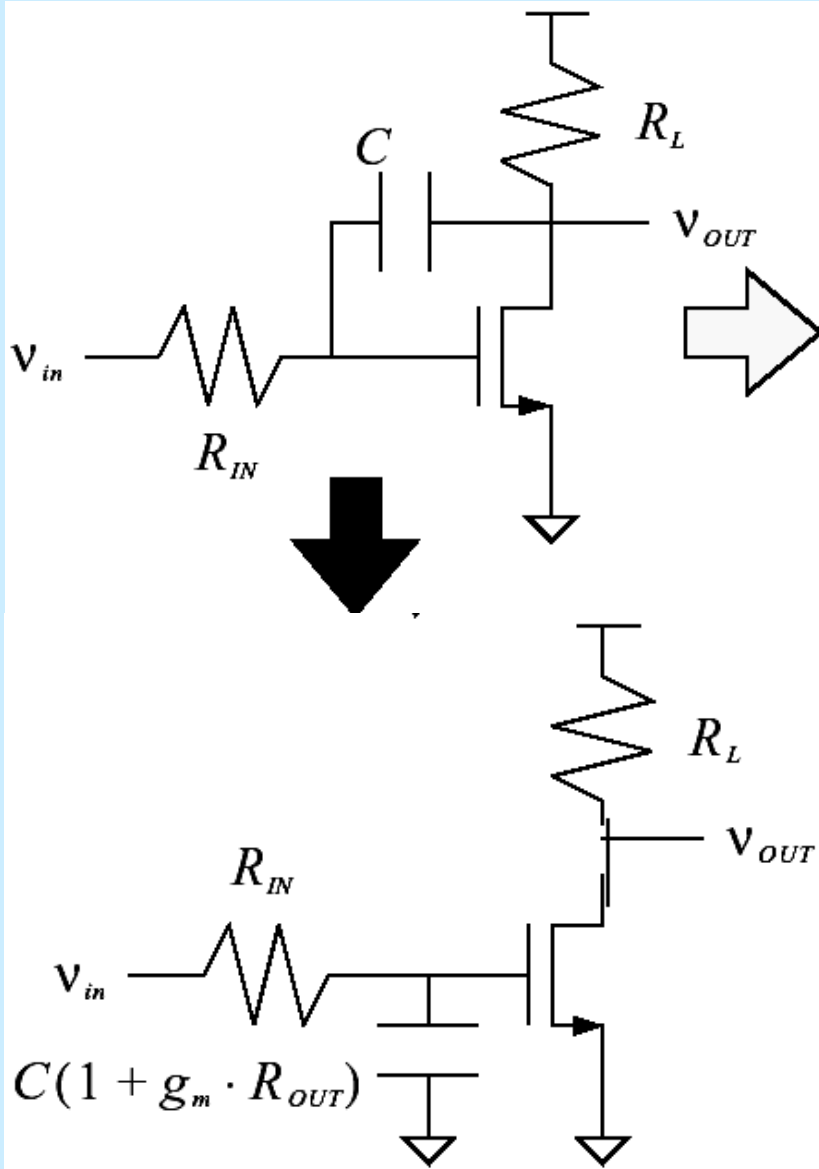
$$i_c = C \cdot \frac{d}{dt}(v_{in} - v_{out})$$

$$i_c = C \cdot \frac{d}{dt}(v_{in} + A \cdot v_{in})$$

$$i_c = C \cdot (1 + A) \cdot \frac{dv_{in}}{dt}$$



Miller approximation – Common source



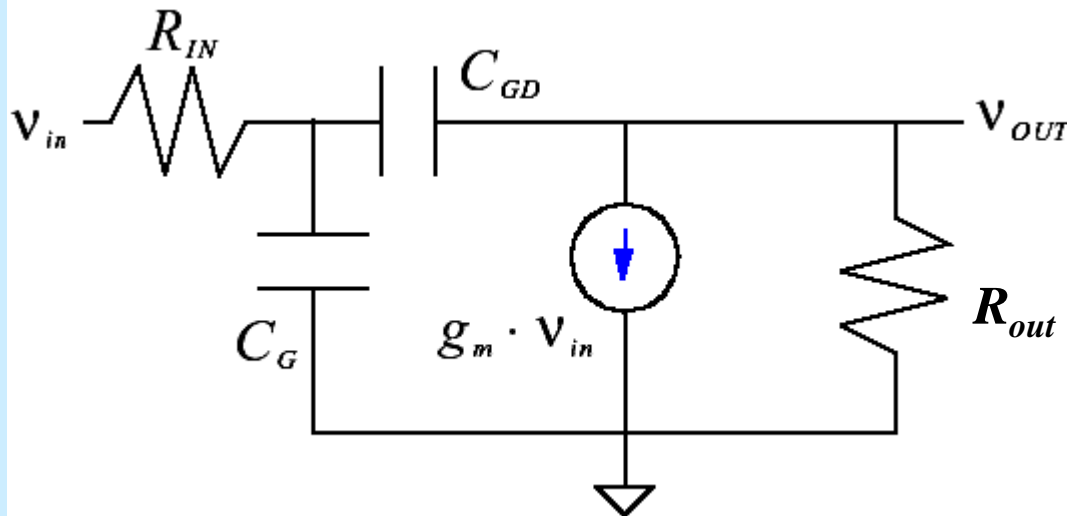
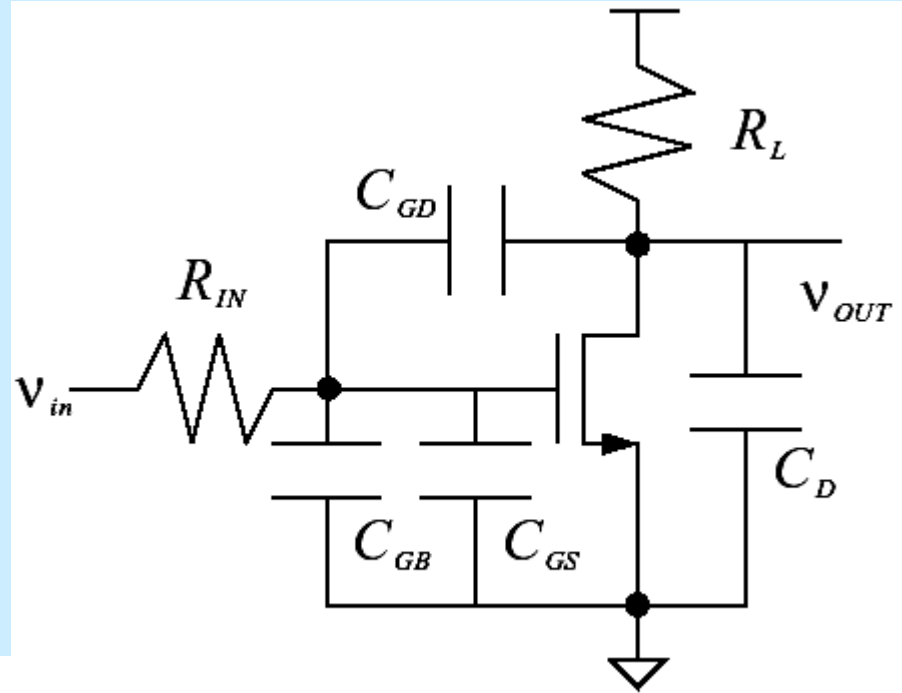
$$\frac{V_{out}}{V_{in}} = \frac{g_m \cdot R_{out}}{1 + j\omega[R_{in}C(1 + g_m \cdot R_{out}) + R_{out}C]}$$

$$\omega_p = \frac{1}{R_{in} \cdot C \cdot (1 + g_m \cdot R_{out})}$$

Miller Capacitor

Common Source

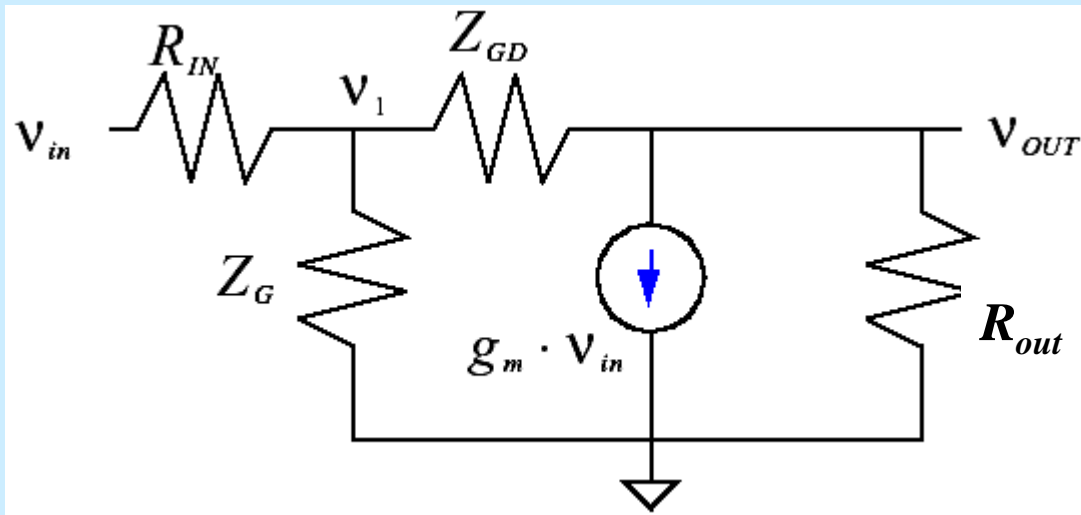
- C_D can be ignored sometimes
- $R_{out} = R_L || r_o$
- $C_G = C_{GB} + C_{GS}$



Common source – small signal

- Using impedances

$$\frac{V_1 - V_{in}}{R_{in}} + \frac{V_1}{Z_G} + \frac{V_1 - V_{out}}{Z_{GD}} = 0$$



$$\frac{V_{out}}{V_{in}} = -g_m \cdot R_{out} \cdot \frac{1 - \frac{C_{GD}}{g_m}}{1 + j\omega \{ [C_{GD}(1 + g_m \cdot R_{out}) + C_G] \cdot R_{in} + R_L C_{GD} \} - \omega^2 R_{out} R_{in} C_G C_{GD}}$$

$$\frac{V_{out}}{V_{in}} = \frac{1 - j\omega \frac{C_{GD}}{g_m}}{\left(1 + j\frac{\omega}{\omega_{p1}}\right) \cdot \left(1 + j\frac{\omega}{\omega_{p2}}\right)} = \frac{1 - j\omega \frac{C_{GD}}{g_m}}{1 + j\omega \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) - \omega^2 \cdot \frac{1}{\omega_{p1}} \cdot \frac{1}{\omega_{p2}}}$$

Common source – Poles and Zeros

- From the transfer function:

$$\omega_{p1} = -\frac{1}{R_{in} \cdot [C_{GD}(1 + g_m \cdot R_{out}) + C_G] + R_L C_{GD}}$$

$$\omega_{p2} = -\frac{1}{R_{out} C_{GD}} - \frac{1}{(R_{out} || R_{in} || \frac{1}{g_m}) C_G}$$

$$\omega_z = \frac{g_m}{C_{GD}}$$

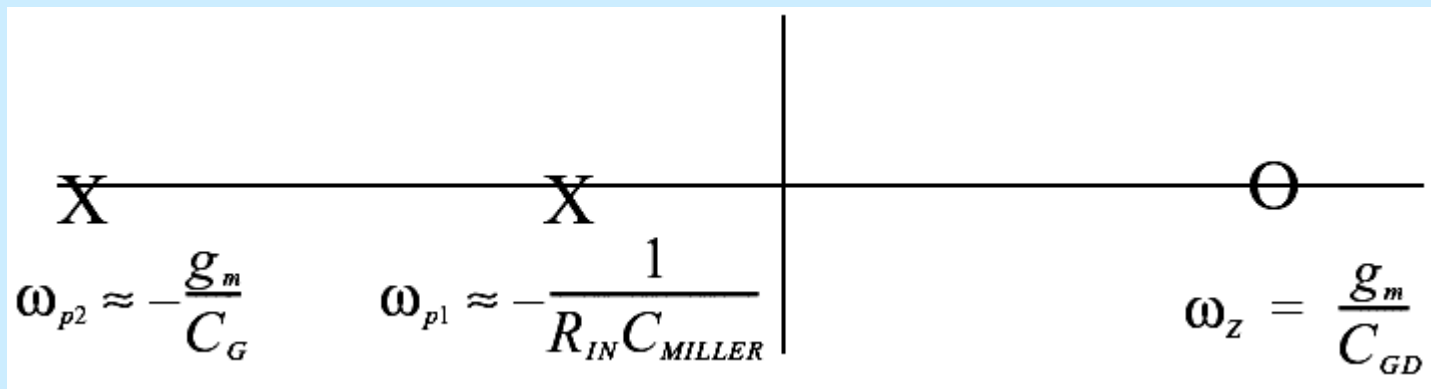
$$H(\omega) = \frac{\left(1 + j \frac{\omega}{\omega_z}\right)}{\left(1 + j \frac{\omega}{\omega_{p1}}\right) + \left(1 + j \frac{\omega}{\omega_{p2}}\right)}$$

Common source – Poles and Zeros

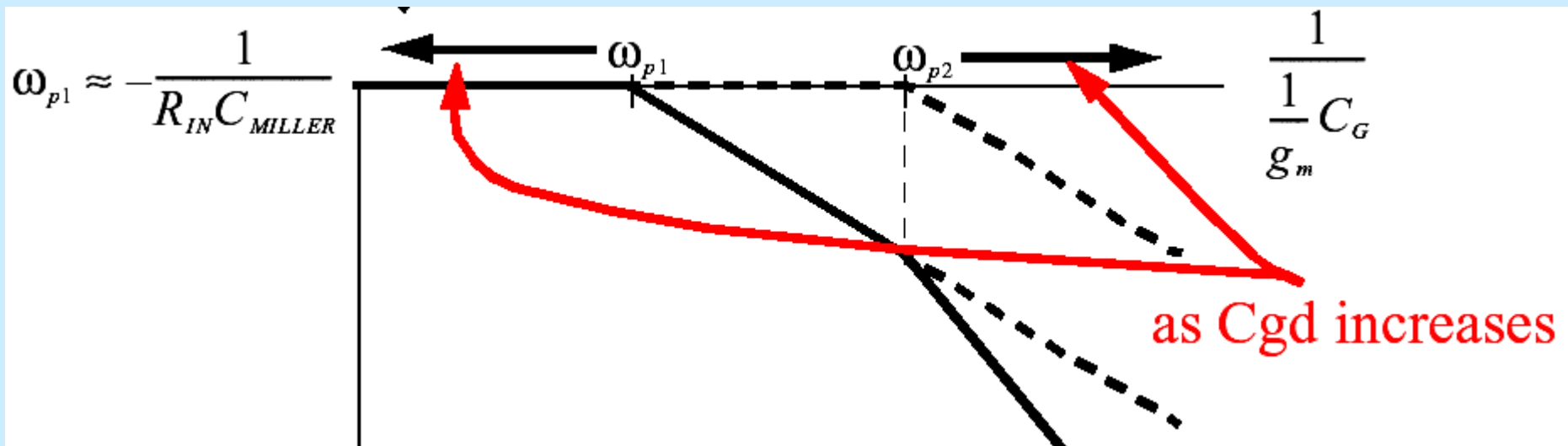
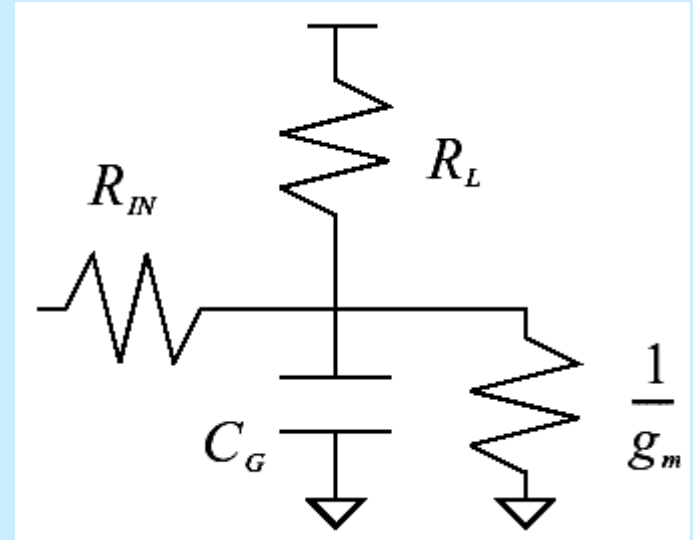
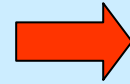
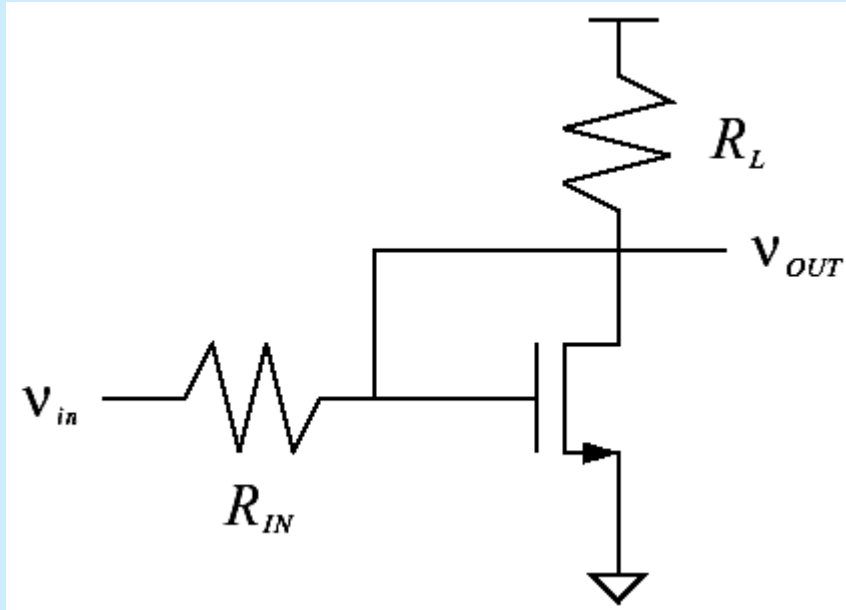
- Converting to s space:

$$s_z = -j\omega_z \quad s_{p1} = -j\omega_{p1} \quad s_{p2} = -j\omega_{p2}$$

$$H(\omega) = \frac{\left(1 + j\frac{\omega}{\omega_z}\right)}{\left(1 + j\frac{\omega}{\omega_{p1}}\right) + \left(1 + j\frac{\omega}{\omega_{p2}}\right)} \quad \rightarrow \quad H(s) = \frac{\left(1 - \frac{s}{s_z}\right)}{\left(1 - \frac{s}{s_{p1}}\right) + \left(1 - \frac{s}{s_{p2}}\right)}$$



Diode connected and Pole Splitting

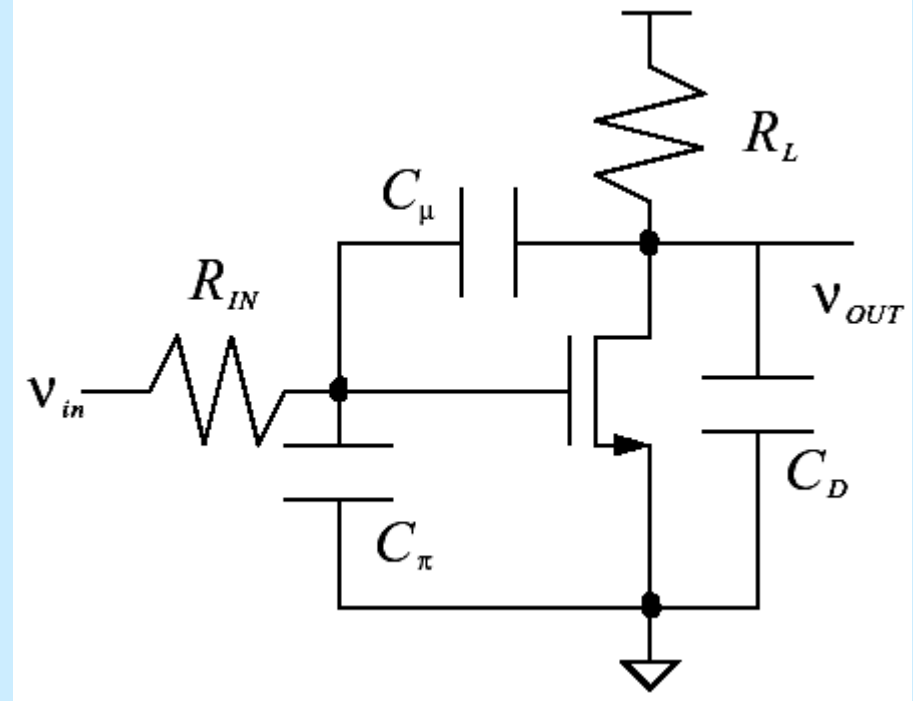


Common source – Capacitance Cases

- Relative magnitude of the capacitors result in different scenarios

- Case1: Miller Cap small

$$R_{in}C_{\pi}, R_{out}C_D \gg R_{in}C_{Miller}$$



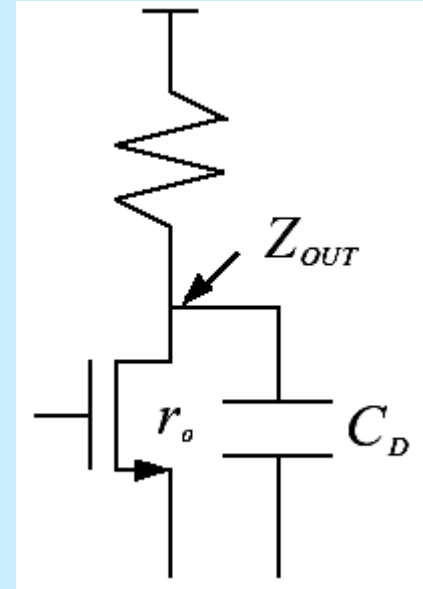
$$\omega_{p1} = \frac{1}{R_{in}C_{\pi}} \quad \omega_{p2} = \frac{1}{R_{out}C_D}$$

$$R_{out} = R_L || r_o$$

Common source – Small Miller capacitance

- Output Impedance, Z_{out}

$$Z_{out} = R_L \parallel r_o \parallel \frac{1}{j\omega C_D} = R_{out} \parallel \frac{1}{j\omega C_D}$$



- Stage gain, A_v

$$A_v = -g_m \cdot Z_{out} = -g_m \cdot \frac{\frac{R_{out}}{j\omega C_D}}{R_{out} + \frac{1}{j\omega C_D}} = \frac{R_{out}}{1 + j\omega R_{out} C_D}$$

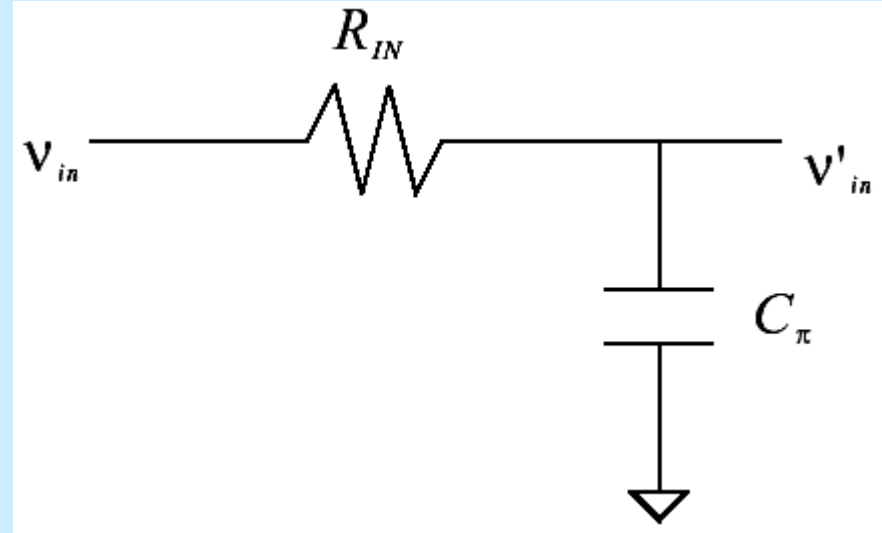
- Output pole

$$\omega_p = \frac{1}{R_{out} C_D}$$

Common source – Small Miller capacitance

- Input transfer function

$$\frac{v'_{in}}{v_{in}} = \frac{1}{R_{in} + \frac{1}{j\omega C_{\pi}}} = \frac{1}{1 + j\omega R_{in} C_{\pi}}$$



- Input pole

$$\omega_p = \frac{1}{R_{in} C_{\pi}}$$

Common source – Other cases

- Case 2: Large C_D

$$R_{\text{out}} C_D \gg R_{\text{in}} C_{\text{Miller}}, R_{\text{in}} C_\pi$$

$$\omega_{p1} = \frac{1}{R_{\text{out}} C_D} \qquad \omega_{p2} = \frac{1}{R_{\text{in}} (C_\pi + C_\mu)}$$

- Case 3: Large C_μ

$$R_{\text{in}} C_{\text{Miller}} \gg R_{\text{out}} C_D, R_{\text{in}} C_\pi$$

$$\omega_{p1} = \frac{1}{R_{\text{in}} \underbrace{(1 + g_m R_{\text{out}}) C_\mu}_{C_{\text{Miller}}}} \qquad \omega_{p2} = \frac{1}{\frac{1}{g_m} (C_\pi + C_D)} = \frac{g_m}{(C_\pi + C_D)}$$

Poles and Zeros

- Usually the multiplying factor on the Miller capacitor results in poles far apart from each other than in other cases.
- The pole splitting is used to compensate the circuit.

$$\omega_{p1} \approx \frac{1}{R_{in} C_{MILLER}}$$

Diagram illustrating the pole splitting technique. The horizontal axis represents the real frequency axis. The poles are marked with 'X' and the zero with 'O'. The poles are located at ω_{p1} and ω_{p2} , while the zero is at ω_z . The diagram shows the effect of the Miller capacitor on the poles, with the poles moving far apart due to the multiplying factor on the Miller capacitor.

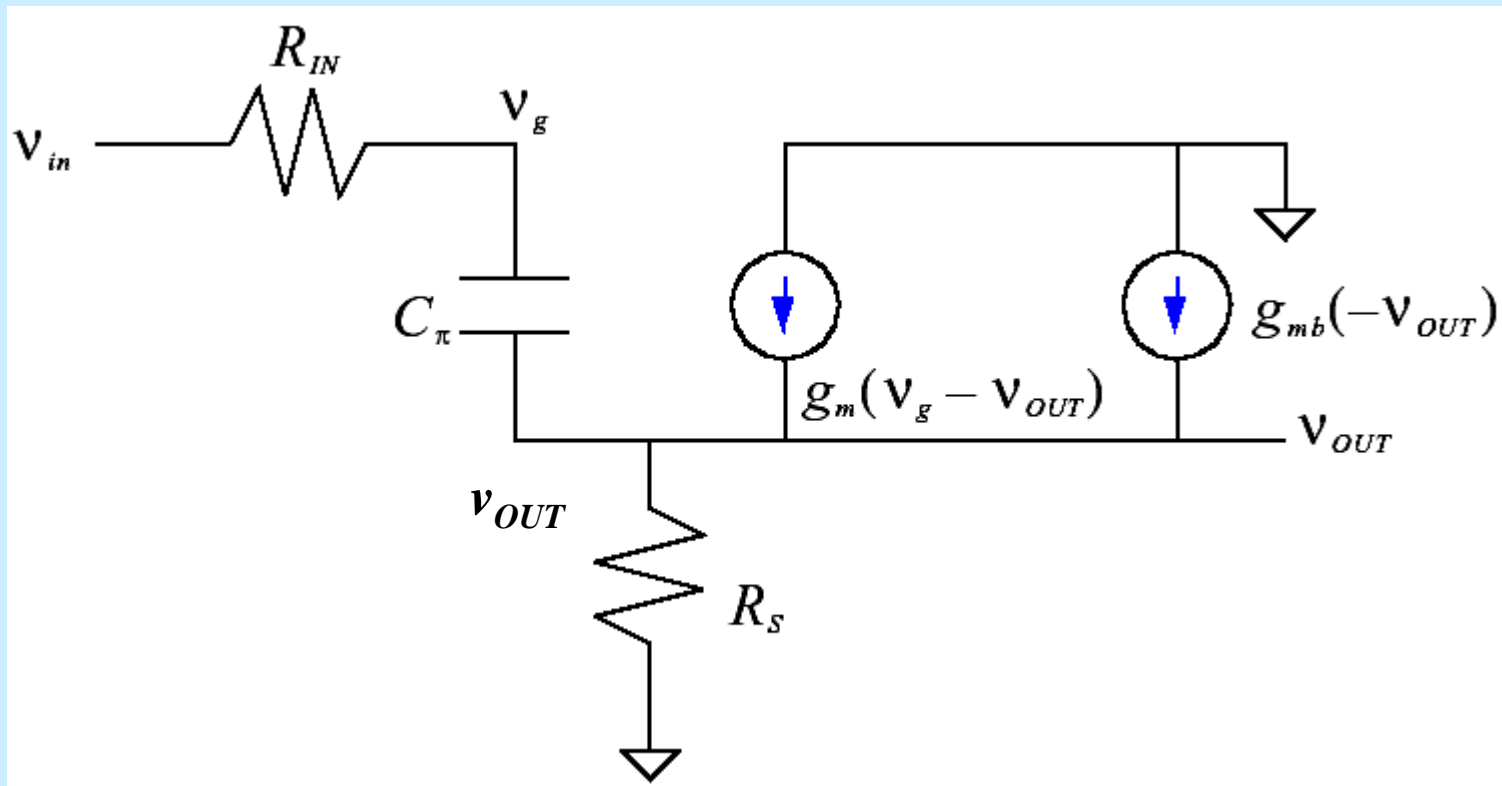
Labels for poles and zero:

- Pole 1 (leftmost): C_μ large, $\omega_{p2} \approx \frac{g_m}{(C_\pi + C_D)}$
- Pole 2: $C_\mu = 0$, $\frac{1}{R_{OUT} C_D}$
- Pole 3: $C_\mu = 0$, $\frac{1}{R_{IN} C_\pi}$
- Pole 4 (rightmost): C_μ large, $\omega_{p1} \approx \frac{1}{R_{in} C_{MILLER}}$
- Zero: $\omega_z = \frac{g_m}{C_\mu}$

Common drain (source follower)

- Small circuit analysis

$$v_g = \left(\frac{1}{1 + j\omega R_{in} C_\pi} \right) \bullet (v_{in} - v_{out}) + v_{out}$$



Common drain – Small signal analysis

$$\frac{V_{\text{out}}}{R_s} = \frac{V_g - V_{\text{out}}}{\frac{1}{j\omega C_\pi}} + g_m \bullet V_g - (1 + \chi) \bullet g_m \bullet V_{\text{out}}$$

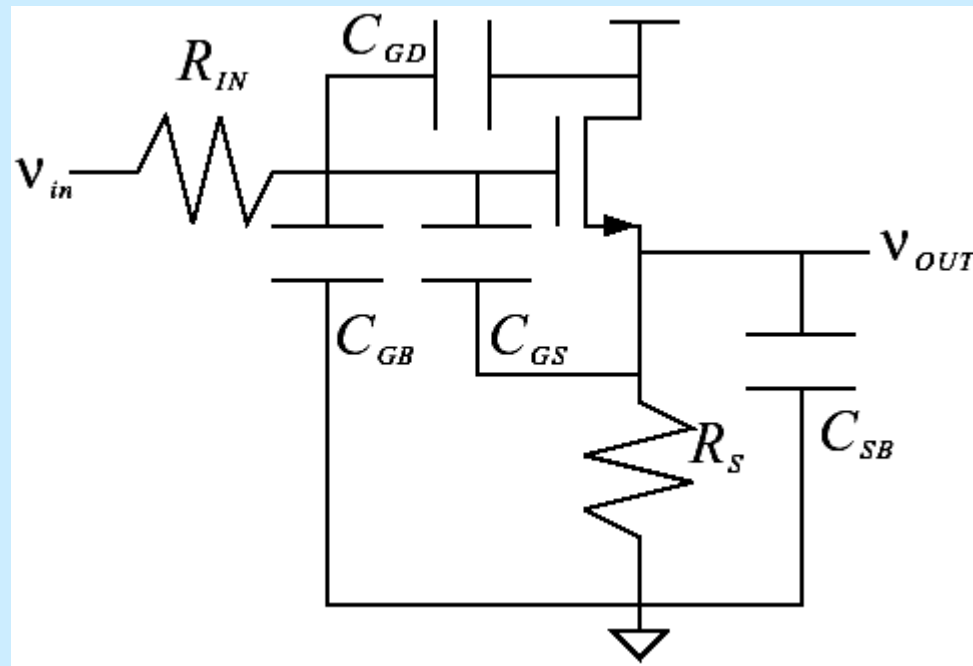
$$V_{\text{out}} \bullet \left(\frac{1}{R_s} + (1 + \chi) \bullet g_m + j\omega C_\pi \right) = (j\omega C_\pi + g_m) \bullet V_g$$

$$\frac{V_{\text{out}}}{R_s} = \frac{g_m R_s}{1 + (1 + \chi) \bullet g_m R_s} \bullet \frac{\left(1 + j\omega \frac{C_\pi}{g_m} \right)}{1 + j\omega R_{\text{in}} C_\pi \bullet \frac{1 + \chi \bullet g_m R_s}{1 + (1 + \chi) \bullet g_m R_s}}$$

$$\omega_z = \frac{g_m}{C_\pi} \quad \omega_{p1} = \frac{1}{R_{\text{in}} C_\pi (1 - A)} \quad A = \frac{g_m R_s}{1 + (1 + \chi) g_m R_s}$$

Common drain (source follower)

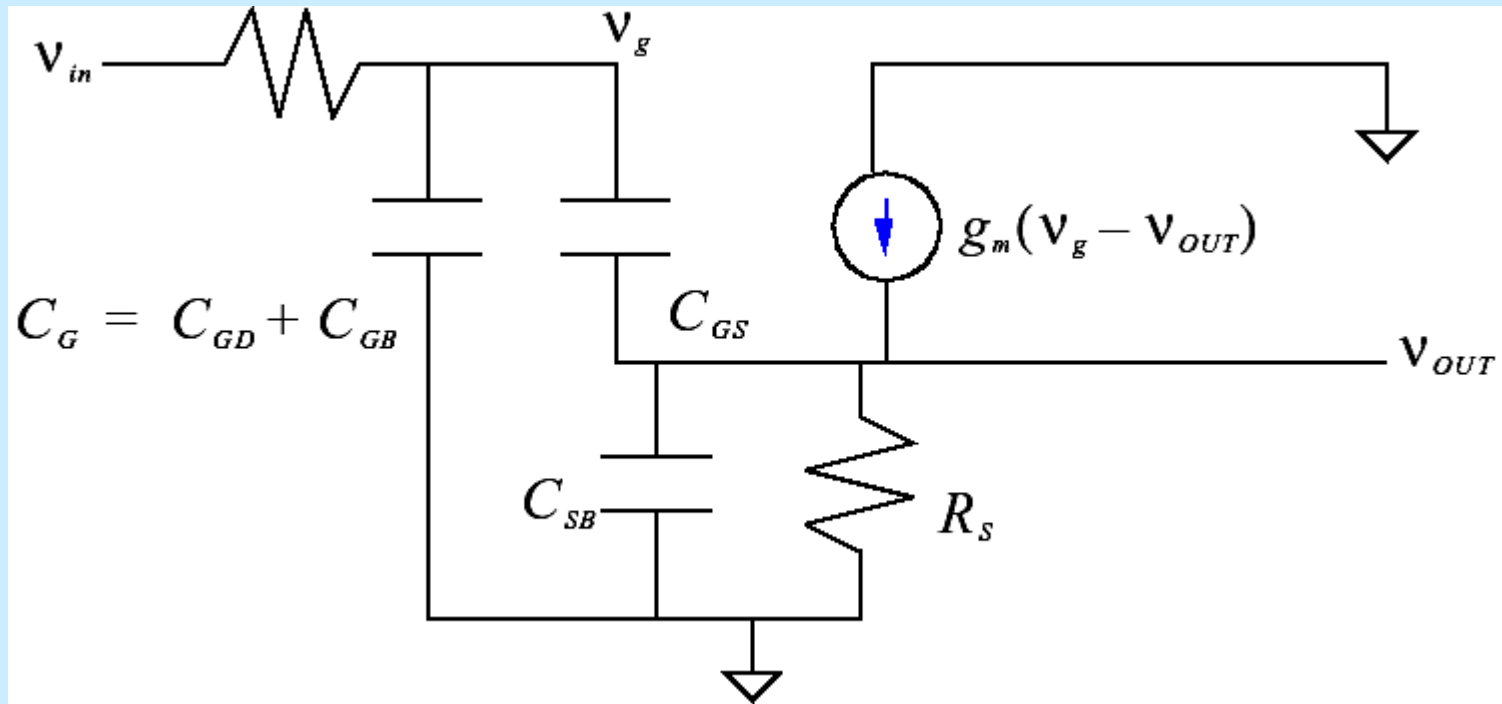
- Effect of C_{SB}
- The Body is Grounded



Common drain – small signal

$$\frac{V_{in} - V_g}{R_{in}} = v_g \cdot j\omega C_G + (v_g - v_{out}) \cdot j\omega C_{GS}$$

$$(v_g - v_{out}) \cdot j\omega C_{GS} - g_m \cdot (v_g - v_{out}) = \frac{V_{out}}{R_s} + v_{out} \cdot j\omega C_{SB}$$



Common drain – Small signal analysis

$$\frac{V_{out}}{V_{in}} = \frac{\frac{g_m R_s}{1 + g_m R_s} \cdot \left(1 + j\omega \frac{C_{GS}}{g_m}\right)}{1 + j\omega \left[R_{in} C_G + \frac{R_{in} C_{GS}}{1 + g_m R_s} + \frac{R_s (C_{GS} + C_{SB})}{1 + g_m R_s} \right] - \omega^2 R_s R_{in} \left[\frac{C_{GS} C_G + C_{SB} (C_G + C_{GS})}{1 + g_m R_s} \right]}$$

- Having the denominator to be in the format:

$$\left(1 + j\frac{\omega}{\omega_{p1}}\right) \cdot \left(1 + j\frac{\omega}{\omega_{p2}}\right) = 1 + j\omega \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \right) - \frac{\omega^2}{\omega_{p1} \omega_{p2}}$$

- The poles are:

$$\omega_{p1} = \frac{1}{R_{in} C_G + \frac{R_{in} C_{GS}}{1 + g_m R_s} + \frac{R_s (C_{GS} + C_{SB})}{1 + g_m R_s}} = \frac{1}{R_{in} C_G + \frac{R_{in} C_{GS}}{1 + g_m R_s} + R_O (C_{GS} + C_{SB})}$$

$$\omega_{p2} = \frac{R_{in} C_G + \frac{R_{in} C_{GS}}{1 + g_m R_s} + R_O (C_{GS} + C_{SB})}{R_O R_{in} [C_{GS} C_G + C_{SB} C_G + C_{SB} C_{GS}]}$$

$$R_O = \frac{1}{g_m} || R_s$$

Common drain - Cases

- Case 1: $R_{in} \left(C_G + \frac{C_{GS}}{1 + g_m R_s} \right) \gg R_O (C_{GS} + C_{SB})$

$$\omega_{p1} = \frac{1}{R_{in} \left(C_G + \frac{C_{GS}}{1 + g_m R_s} \right)}$$

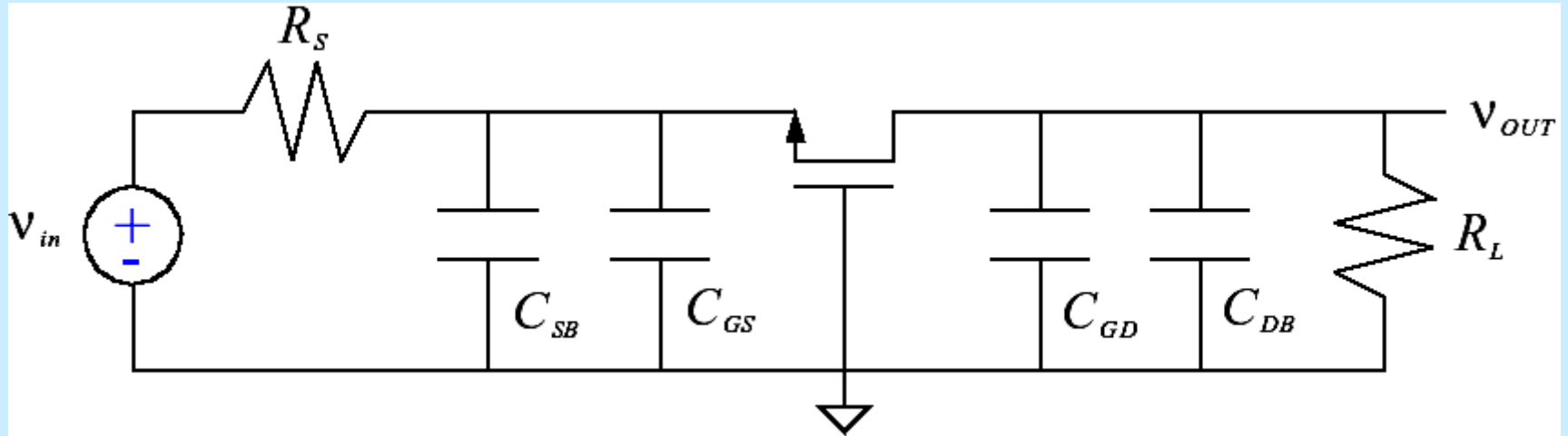
$$A = \frac{g_m R_s}{1 + g_m R_s}$$

$$C_{\text{Miller}} = C_{GS} \cdot (1 - A)$$

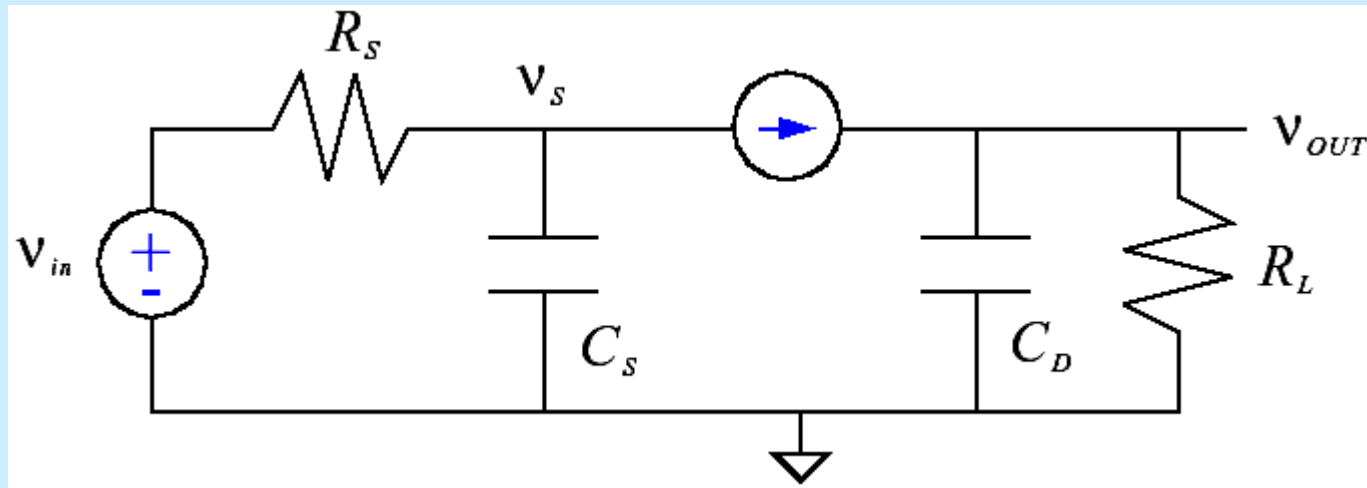
- Case 2: $R_O (C_{GS} + C_{SB}) \gg R_{in} \left(C_G + \frac{C_{GS}}{1 + g_m R_s} \right)$

$$\omega_{p1} = \frac{1}{R_O (C_{GS} + C_{SB})}$$

Common Gate



- Assuming $r_o \rightarrow \infty$



Common gate - small signal

- Using KCL @ v_s and @ v_{out}

$$\frac{V_{in} - V_s}{R_s} = V_s \cdot j\omega C_s + g_m V_s$$

$$g_m V_s = v_{out} \cdot j\omega C_D + \frac{v_{out}}{R_L}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{g_m R_L}{1 + g_m R_s}}{(1 + j\omega R_L C_D) \cdot \left(1 + j\omega \frac{R_s C_s}{1 + g_m R_s}\right)}$$

- No Zeros

$$\omega_{p1} = \frac{1}{R_L C_D}$$

$$\omega_{p2} = \frac{1}{\frac{R_s}{1 + g_m R_s} C_s} = \frac{1}{\left(R_s \parallel \frac{1}{g_m}\right) C_s}$$