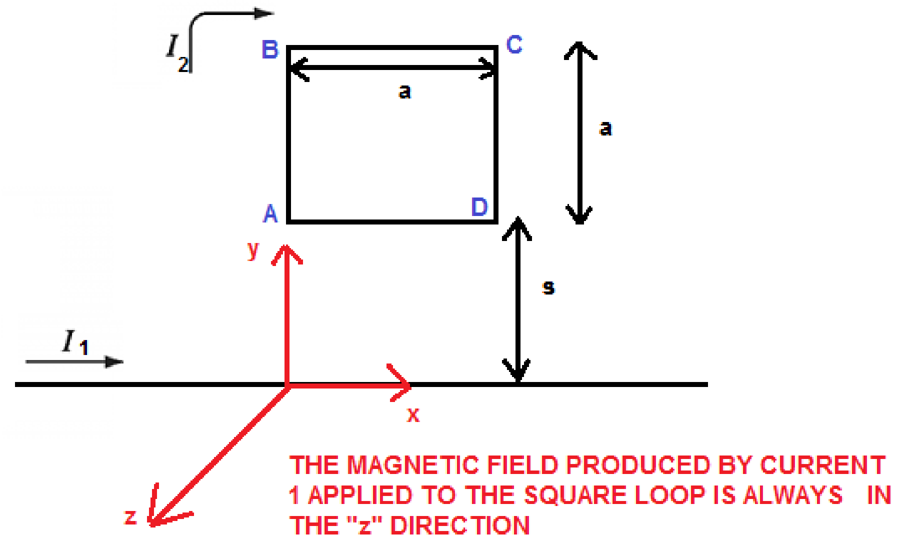
Calculus of the force applied on a square loop due to a infinite straight wire



One should remember the formula of the Force that appears on a wire inserted on a magnetic field which is as follows:

$$\vec{F} = I \cdot \int \overrightarrow{u_T} x \vec{B} \cdot dl$$

I=current flowing in the wire; **u_T**=tangential vector in the direction of the current

B=external magnetic field vector applied to the wire

The magnetic field produced by a straight wire is as follows:

$$\vec{B} = \frac{\mu_0 \cdot I}{2\pi \cdot d} \cdot \overrightarrow{u_\theta}$$

u_θ=vector tangent to the circumference around the straight wire (right hand rule)

d=distance from the straight wire to the point you want to find the B field

I= current through the straight wire

Let's start to calculate what is needed:

Force applied to the D-A wire due to the straight wire:

$$\overrightarrow{F_{DA}} = I_2 \int_a^0 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 0 & \frac{\mu_0 \cdot I_1}{2\pi \cdot s} \end{vmatrix} \cdot dl = \frac{\mu_0 \cdot I_1 \cdot I_2}{2\pi \cdot s} \cdot a \cdot \vec{j} \ (repelling)$$

Force applied to the A-B wire due to the straight wire:

$$\overrightarrow{F_{AB}} = I_2 \int_s^{s+a} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\mu_0 \cdot I_1}{2\pi \cdot l} \end{vmatrix} \cdot dl = \frac{\mu_0 \cdot I_1 \cdot I_2}{2\pi} \cdot \vec{i} \cdot \int_s^{s+a} \frac{1}{l} dl = \frac{\mu_0 \cdot I_1 \cdot I_2}{2\pi} \cdot \ln\left(\frac{s+a}{s}\right) \cdot \vec{i}$$

Force applied to the B-C wire due to the straight wire: I leave this to you

Force applied to the C-D wire due to the straight wire: I leave this to you