

From first principles proof that the DTFT of the sequence;

$$x_2[n] = -\alpha^n \mu[-n-1], \quad |\alpha| > 1 \text{ is } \frac{1}{1 - \alpha e^{-j\omega}}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} -\alpha^n \mu[-n-1] e^{-j\omega n}$$

$$X(e^{j\omega}) = - \sum_{n=-\infty}^{\infty} \alpha^n \mu[-n-1] e^{-j\omega} e^n$$

$$X(e^{j\omega}) = -e^{-j\omega} \sum_{n=-\infty}^{\infty} \alpha^n \mu[-n-1] e^n$$

$$= -e^{-j\omega} \sum_{n=-\infty}^{-1} \alpha^n e^n$$

$$= -e^{-j\omega} \sum_{n=1}^{\infty} (\alpha e)^{-n}$$

$$= -e^{-j\omega} \left( \left( \frac{1}{\alpha e} \right)^0 + \sum_{n=0}^{\infty} \left( \frac{1}{\alpha e} \right)^n \right)$$

$$= -e^{-j\omega} - e^{-j\omega} \sum_{n=0}^{\infty} \left( \frac{1}{\alpha e} \right)^n$$

$$= -e^{-j\omega} + \frac{-e^{-j\omega}}{1 - \frac{1}{\alpha e}}$$