

From first principles proof that the DTFT of the sequence;

$$x_2[n] = -\alpha^n \mu[-n-1], \quad |\alpha| > 1 \text{ is } \frac{1}{1 - \alpha e^{-j\omega}}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} -\alpha^n \mu[-n-1] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-1} -\alpha^n e^{-j\omega n}$$

$$= \sum_{n=1}^{\infty} -\alpha^{-n} e^{j\omega n}$$

$$= -\alpha^0 e^{j\omega 0} + \sum_{n=0}^{\infty} -\alpha^{-n} e^{j\omega n}$$

$$= -1 + \sum_{n=0}^{\infty} -\left(\frac{1}{\alpha}\right)^n e^{j\omega n}$$

$$= -1 - \frac{1}{1 - \frac{1}{\alpha} e^{-j\omega n}}$$