RESPONSE OF LTI SYSTEM TO A COMPLEX EXPONENTIA

INPUT TO LTI SYSTEM

$$x[n] = z^n$$

$$z = re^{j\omega}$$

COMPLEX EXPONENTIAL

IF $x[n] \xleftrightarrow{FS}$ IS a_k, AND $y[n] \xleftrightarrow{FS} b_k$

$$\underbrace{b_k}_{\text{OUTPUT}} = \underbrace{H(e^{jk\omega_0})}_{\text{FREQ. RESPONSE}} \cdot \underbrace{a_k}_{\text{INPUT}}$$

OUTPUT = FREQ. RESPONSE · INPUT

???

(i) FIND $H(z)$

(ii) FIND $a_k \xleftrightarrow{FS} x[n]$

(iii) FIND $b_k = H(e^{jk\omega_0})a_k$

(iiii) FIND $y[n] \xleftrightarrow{FS} b_k$

$x(n) \rightarrow \boxed{h(n)} \xrightarrow{y(n)}$ $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = Z\{x[n]\}$

WHERE $z = re^{j\omega}$

HW#4

$h[n] = 0$ for $n < 0$

① $y[n] + 2y[n-2] = x[n] - x[n-3]$; SYSTEM W/ INITIAL REST

(a) FIND $h[n]$; IF $x[n] = \delta[n]$, THEN $y[n] = h[n]$

$\sqrt{2j}$

$\therefore$ $h[n] = \delta[n] - \delta[n-3] - 2h[n-2]$

$h[0] = \delta[0] - \cancel{\delta[-3]}^{0} - \cancel{2h[-2]}^{0} = 1 \quad = 1 = h[0]$ 5

$h[1] = \cancel{\delta[1]}^{0} - \cancel{\delta[-2]}^{0} - \cancel{2h[-1]}^{0} = 0 \quad = 0 = h[1]$ -2

$h[2] = \delta[2]^{0} - \delta[-1]^{0} - 2h[0] = 0 - 2(1) = -2 = h[2]$

$h[3] = \delta[3]^{0} - \delta[0] - \cancel{2h[1]}^{0} = 0 - (1) = -1 = h[3]$

$h[4] = -2h[2] = -2(-2) = 4 = h[4]$

$h[5] = -2h[3] = -2(-1) = 2 = h[5]$

$h[6] = -2h[4] = -2(4) = -8 = h[6]$

$h[7] = -2h[5] = -2(2) = -4 = h[7]$

$h[8] = -2h[6] = -2(-8) = 16 = h[8]$

$h[9] = -2h[7] = -2(-4) = 8 = h[9]$

$h[10] = -2h[8] = -2(16) = -32 = h[10]$

-16

$x'[n] = \delta[n] - \delta[n-3]$

$h_1[n]$

$h_2[n]$

$2(n-2)$

$\boxed{h = h_1 + h_2}$

$h_1[n] = -\delta[n] - 2h_1[n-2]$

$h_1[0] = \delta[0]$

$h_1[1] = 0$

$h_1[2] = -2h[0] = -2$

$h_1[3] = -2h[1] = 0$

$h_1[4] = -2h[2] = \quad 2\times 2$

$h_1[5] = -2h[3] =$

$h_1[6] = -2h[4] = \quad 4\times 2$

$h_1[7] = -2h[5] =$

$h_1[8] = -2h[6] = \quad 8\times 2$

$h_1\downarrow$		$h_2\downarrow$		$h\downarrow$
= 1	+	0	=	1
= 0	+	0	=	0
= -2	+	0	=	-2
= 0	+	-1	=	-1
= 4	+	0	=	4
= 0	+	-2	=	2
= -8	+	0	=	-8
= 0	+	-4	=	-4
= 16	+	0		16
0				

$h_2[n] = -\delta[n-3] - 2h_2[n-2]$

$h_2[0] = 0$ $\qquad = 0$

$h_2[1] = 0$ $\qquad = 0 \qquad 2$

$h_2[2] = 0$ $\qquad = 0$

$h_2[3] = -1$ $\qquad = -1$

$h_2[4] = -2h[2] = 0$ $\qquad = 0$

$h_2[5] = -2h[3] = 2$ $\qquad = 2$

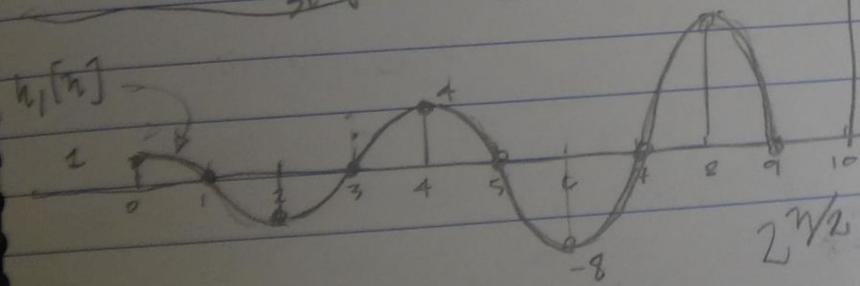
$h_2[6] = -2h[4] = 0$ $\qquad = 0$

$h_2[7] = -2h[5] = 4$ $\qquad = -4$

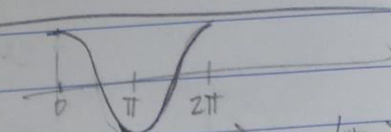
$h_3[8] = -2h[6] = 0$ $\qquad = 0$

$h_3[9] = -2h[7] =$ $\qquad = 8$

$h_1[n]$



$\cos(x) =$



$\cos(\omega t)$

$\cos(2\pi f t) = \cos\left(\frac{2\pi}{T} t\right) = \cos\left(\frac{\pi}{2} t\right)$

$(T = 4) \quad n$

$2^{n/2} \cos\left(\frac{\pi}{2} n\right) = h_1[n]$

1a $\quad z = re^{j\omega}$

$h[n] =$

$$2^{n/2}\cos\left(\tfrac{\pi}{2}n\right)u(n) - 2^{\left(\frac{n-3}{2}\right)}\cos\left(\tfrac{\pi}{2}(n-3)\right)u(n-3)$$

(b) $Z\{y[n] + 2y[n-2] = x[n] - x[n-3]\}$

$$= Y(z) + 2Y(z)z^{-2} = X(z) - X(z)z^{-3}$$

$$= Y(z)(1 + 2z^{-2}) = X(z)(1 - z^{-3})$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \left(\frac{1-z^{-3}}{1+2z^{-2}}\right)\left(\frac{z^3}{z^3}\right) = \frac{z^3-1}{z^3+2z} = H(z)$$

(c) FROM TABLE $Z\{\cos \omega n\} = \dfrac{(1 - z^{-1}\cos \omega n)}{(1 - 2z^{-1}\cos\omega n + z^{-2})}$

$$Y(z) = \frac{(1 - z^{-1}\cos\omega n)(z^3 - 1)}{(1 - 2z^{-1}\cos\omega n + z^{-2})(z^3 + 2z)}$$

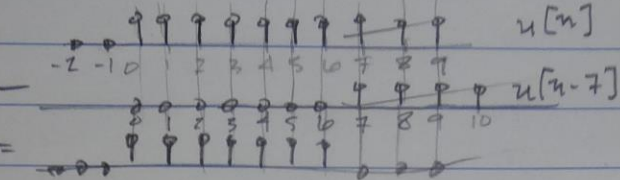
I CANNOT MULTIPLY DENOMINATOR AND THEN FACTOR TO PERFORM PARTIAL FRACTION DECOMP'N SO I CAN THEN DO INVERSE Z-T'FORM

② $h[n] = \begin{cases} 1 & 0 \le n \le 6 \\ 0 & \text{ELSE} \end{cases}$

$$\boxed{H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}}$$

$$\boxed{z = re^{j\omega}}$$

(a) $h[n] = u[n] - u[n-7]$



$Z\{h[n]\} = Z\{u[n]\} - Z\{u[n-7]\}$

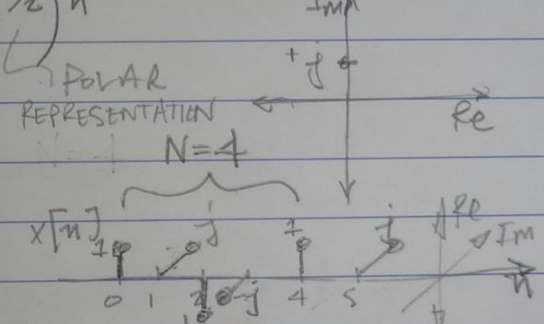
$$\boxed{X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}} = \sum_{n=0}^{6} x[n] z^{-n}$$

$$\boxed{\sum_{n=0}^{N} x^n = \frac{1-x^{N+1}}{1-x}}$$

$X(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}$

$X(z) = \frac{1-z^7}{1-z}$

(b) $x[n] = (-1)^{(n/2)} = \left(-1^{(1/2)}\right)^n = (j)^n = \left(e^{j\pi/2}\right)^n$



$$x[n] \xleftrightarrow{FS} \boxed{a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-j\frac{2\pi k n}{N}}}$$

$a_k = \left(\frac{1}{4}\right) \sum_{n=0}^{4} e^{j\frac{2\pi n}{4}} \cdot e^{-j\frac{2\pi k n}{4}}$

$= \left(\frac{1}{4}\right) \sum_{n=0}^{4} \left(e^{j\frac{2\pi}{4}(1-k)}\right)^n ; A = e^{j\frac{2\pi}{4}(1-k)}$

$= \left(\frac{1}{4}\right) \sum_{n=0}^{4} A^n \Rightarrow \boxed{\sum_{n=0}^{N} A^n = \frac{1-A^{N+1}}{1-A}}$

$\Rightarrow a_k = \left(\frac{1}{4}\right) \frac{1-A^5}{1-A} = \left(\frac{1}{4}\right) \frac{1-e^{j\frac{5\pi}{2}(1-k)}}{1-e^{j\frac{\pi}{2}(1-k)}} = a_k$

$$\boxed{b_k = H(e^{jk\omega_0}) a_k} = \left(\frac{1}{4}\right)\left(\frac{1-z^7}{1-z}\right)\left(\frac{1-e^{j\frac{5\pi}{2}(1-k)}}{1-e^{j\frac{\pi}{2}(1-k)}}\right)$$

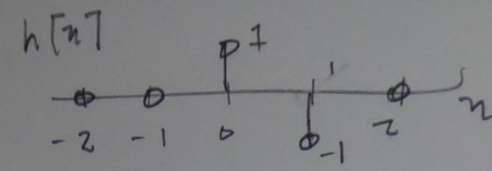
② CONT'D

$$\omega_0 = \frac{2\pi}{N}$$

$$b_k = \left(\frac{1}{4}\right)\left(\frac{1 - e^{(jk\omega_0 7)}}{1 - e^{(jk\omega_0 7)}}\right)\left(\frac{1 - e^{j\frac{5\pi}{2}(1-k)}}{1 - e^{j\pi/2(1-k)}}\right)$$

$$= \frac{1 - e^{j\frac{5\pi}{2}(1-k)} - e^{jk} + e}{1 -}$$

(3) $h[n] = \begin{cases} 1 & n=0 \\ -1 & n=1 \\ 0 & \text{ELSE} \end{cases}$

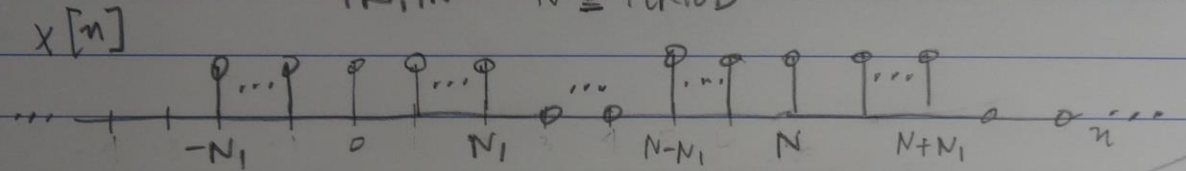


$$H(z) = \sum_{n=0}^{\infty} h[n] z^{-n} = \sum_{n=0}^{1} h[n] z^{-n}$$

(a) $= 1\cdot 1 + -1\, z^{-n} = 1 - z^{-n} = H(z)$

(b) $\underbrace{x[n] = \Pi_{N_1, N}[n]}_{\text{PULSE TRAIN}}$; $a_k \xleftrightarrow{FS} x[n]$

PULSE TRAIN $\Rightarrow$ $N_1 \equiv$ PULSE WIDTH/2, $N \equiv$ PERIOD



FROM LECTURE NOTES

$$a_k = \begin{cases} \left(\frac{1}{N}\right) \dfrac{\sin\left(\pi k (2N_1 + 1)/N\right)}{\sin\left(\pi k / N\right)} & k \neq 0 \\ \dfrac{2N_1+1}{N} & k = 0 \end{cases}$$

$$\boxed{b_k = H(e^{jk\omega_0})\, a_k}$$

$b_k = \{$