

Notations

i_D, v_D	Total instantaneous values
I_D, V_D	dc values
i_d, v_d	Total instantaneous ac values
I_d, V_d	Phasor values
$v_D = V_D + v_d$	

Diode

The DC model and equations describe the DC behavior of the device.

$$I_D = I_S \left[e^{\left(\frac{V_D}{nV_T}\right)} - 1 \right] \text{ (Large Signal Equation at DC to find bias point)}$$

Now I have got to analyze the AC behavior. The same form of equation, similar to DC equation above, can be used to describe the AC behavior **[with inclusion of parasitic and inductive effects of the device and the circuit (due to presence of AC) later on]** as in:

$$i_D = I_S \left[e^{\left(\frac{v_D}{nV_T}\right)} - 1 \right] \text{ (Equation to describe complete large signal behavior of device in DC and in AC)}$$
$$(v_D = V_D + v_d)$$

When I use the above large signal equation, just as they are, in the analysis of the device in the AC domain, the AC analysis is large signal AC analysis and is more accurate. This kind of behavior is more general and can be used in all amplifiers (small signal amplifiers and large signal amplifiers) but more needed in large signal amplifiers. When the AC component v_d is small and I linearize the above equation of diode current (shown below) and use the linearized equations to analyze the behavior of the device in the AC domain (including capacitive and inductive effects of the device and the circuit, of course), the AC analysis becomes small signal AC analysis. This is an approximated (and linear) model. It can be used for both large and small signal amplifiers but used mostly in small signal amplifiers where accuracy is not that important.

1) Diode Large Signal DC Model

$$I_D = I_S \left[e^{\left(\frac{V_D}{nV_T}\right)} - 1 \right]$$

2) Diode Large Signal AC Model **Under Forward Bias**

$$i_D = I_S \left[e^{\left(\frac{v_D}{nV_T}\right)} - 1 \right]$$

$$v_D = V_D + v_d$$

$$\Rightarrow i_D = I_S \left[e^{\left(\frac{V_D + v_d}{nV_T}\right)} - 1 \right]$$

$$\Rightarrow i_D \cong I_S e^{\left(\frac{V_D + v_d}{nV_T}\right)}$$

$$\Rightarrow i_D = I_S \left[e^{\left(\frac{V_D}{nV_T}\right)} \right] \cdot \left[e^{\left(\frac{v_d}{nV_T}\right)} \right]$$

$$I_D = I_S \left[e^{\left(\frac{V_D}{nV_T}\right)} - 1 \right]$$

$$\Rightarrow I_D \cong I_S \left[e^{\left(\frac{V_D}{nV_T}\right)} \right]$$

$$\Rightarrow i_D = I_D \cdot \left[e^{\left(\frac{v_d}{nV_T}\right)} \right]$$

This model provides accurate ac analysis and does not neglect non-linear terms. Small signal model is just an approximation of this model.

The relationship between v_d and i_d is non-linear. The v_d is large signal ac voltage.

$v_d \ll V_T$ is not applicable.

~~$$v_d \ll V_T$$~~

For, $v_D = V_D + v_d$

$$i_D \neq I_D + i_d$$

Hence non-linear.

Also no linear relationship between i_d and v_d .

Due to non-linearity in large signal model, superposition theorem can't be applied ($i_D \neq I_D + i_d$). Total response (in the form of current) is not the sum of dc response and ac response for the given dc input and ac input respectively for **non-linear systems**.

3) Diode Small Signal (or Incremental) AC Model Under Forward Bias

$$i_D = I_S \left[e^{\left(\frac{v_D}{nV_T}\right)} - 1 \right]$$

$$v_D = V_D + v_d$$

$$\Rightarrow i_D = I_S \left[e^{\left(\frac{V_D + v_d}{nV_T}\right)} - 1 \right]$$

$$\Rightarrow i_D \cong I_S e^{\left(\frac{V_D + v_d}{nV_T}\right)}$$

$$\Rightarrow i_D = I_S \left[e^{\left(\frac{V_D}{nV_T}\right)} \right] \cdot \left[e^{\left(\frac{v_d}{nV_T}\right)} \right]$$

$$I_D = I_S \left[e^{\left(\frac{V_D}{nV_T}\right)} - 1 \right]$$

$$\Rightarrow I_D \cong I_S \left[e^{\left(\frac{V_D}{nV_T}\right)} \right]$$

$$\Rightarrow i_D = I_D \cdot \left[e^{\left(\frac{v_d}{nV_T}\right)} \right]$$

If the ac signal is small then $v_d \ll V_T$ (v_d is small signal ac voltage) and we can expand the exponential function into a linear series as follows:

$$e^{\left(\frac{v_d}{nV_T}\right)} \cong 1 + \frac{v_d}{nV_T}$$

(higher order terms neglected and only **linear term** retained)

$$\Rightarrow i_D = I_D \left(1 + \frac{v_d}{nV_T}\right) = I_D + \frac{I_D}{nV_T} \cdot v_d = I_D + i_d$$

For a linear system,

$$ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t) \text{ (additivity and homogeneity)}$$

$$\Rightarrow x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t) \text{ (a=1, b=1)}$$

So

$$v_D = V_D + v_d$$

$$\Rightarrow i_D = I_D + i_d$$

Hence the model is indeed linear.

Also the relationship between v_d and i_d is linear

$$i_d = g_d \cdot v_d = \left(\frac{I_{DQ}}{V_T}\right) \cdot v_d$$

Due to linearity in small signal model, the superposition theorem can be applied (and can be used to i_D). Total response (in the form of current) is the sum of dc response and ac response for the given dc input and ac input respectively for **linear systems (superposition theorem)**.