Consider an arbitrary number N

$$N = b_n 2^n + b_{n-1} 2^{n-1} + b_{n-2} 2^{n-2} + \cdots + b_0 2^0$$

Where b can be 0 or 1

$$\left(\frac{N}{5}\right) = \left(\frac{b_n 2^n}{5} + \frac{b_{n-1} 2^{n-1}}{5} + \frac{b_{n-2} 2^{n-2}}{5} + \cdots + \frac{b_0 2^0}{5}\right)$$

The numerators term can be expressed as

$$Numerator = 5x + r$$

Where x is the quotient and r is the remainder when the numerator is divided by 5

Substitution of numerators gives

$$\left(\frac{N}{5}\right) = \left[\frac{b_n(5x_n + r_n)}{5} + \frac{b_{n-1}(5x_{n-1} + r_{n-1})}{5} + \frac{b_{n-2}(5x_{n-2} + r_{n-2})}{5} + \cdots + \frac{b_0(5x_0 + r_0)}{5}\right]$$

Rearranging

$$\left(\frac{N}{5}\right) = \left[\frac{5(b_n x_n + b_{n-1} x_{n-1} + \cdots + b_0 x_0)}{5}\right] + \left[\frac{b_n r_n + b_{n-1} r_{n-1} + \cdots + b_0 r_0}{5}\right]$$

The first bracket is divisible by 5, we don't know about the second bracket. If the second bracket i.e. Sum of remainders is also divisible by 5 then our number N is divisible by 5. For the next iteration, the second bracket becomes our new N. And we express it as a sum of factor and remainder again. We do this over and again till we come to an end. **Note that we are not performing actual division**. The algorithm to express a binary number in terms of factor and remainder when divided by 5 is now discussed. Consider the following

The remainder when 2^n is divided by 5 is shown below

$$2^0 = 4$$
$$2^1 = 3$$
$$2^2 = 1$$
$$2^3 = 2$$
$$2^4 = 4$$
$$2^5 = 3$$
$$2^6 = 1$$
$$2^7 = 2$$

This is an arithmetic progression of powers that give a particular remainder. An expression for powers that give a particular remainder can be written as follows

$$n = 4x = 4$$
$$n = 4x + 1 = 3$$
$$n = 4x + 2 = 1$$
$$n = 4x + 3 = 2$$

From the above four equations, we can get to know the remainder when 2^n is divided by 5 (without actual division)

Example: To determine if 23 is divisible by 5

$23 = 2^4 + 2^2 + 2^1 + 2^0$

$r = (4 + 1 + 3 + 4) = 12$

$12 = 2^3 + 2^2$

$r = (2 + 1) = 3$ Therefore not divisible. If r = 5 at any stage, the number is divisible.