Analysis of the Error Performance of Adaptive Array Antennas for CDMA with Noncoherent M-ary Orthogonal Modulation in Nakagami Fading

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***Abstract*— This letter presents an analytical model for evaluating the Bit Error Rate (BER) of a Direct Sequence Code Division Multiple Access (DS-CDMA) system, with M-ary orthogonal modulation and noncoherent detection, employing an array antenna operating in a Nakagami fading environment. An expression of the Signal to Interference plus Noise Ratio (SINR) at the output of the receiver is derived, which allows the BER to be evaluated using a closed form expression. The analytical model is validated by comparing the obtained results with simulation results.**

***Index Terms*— Adaptive arrays, code division multiple access, bit error rate, Nakagami fading channels, M-ary orthogonal modulation.**

I. Introduction

It IS well known that array antennas can improve the performance of Direct Sequence Code Division Multiple Access (DS-CDMA) systems by reducing the Multiple Access Interference (MAI). An exact analysis of the BER of array antennas for CDMA systems is difficult. Thus approximate analysis methods have been proposed. Recently in [1], a simple analytical method was described to analyse the performance of a DS-CDMA system employing array antennas.

In this letter we use the method presented in [1] to analyse the performance of CDMA, with noncoherent M-ary orthogonal modulation, employing a receiving adaptive antenna in a Nakagami fading environment. It has to be noted that the performance of M-ary orthogonal modulation, which is used in the reverse links of IS-95 and cdma2000 (radio configurations 1 and 2) [2], has been studied extensively in [3] and [4]. However the considerations were restricted to the case of single antenna receivers. Analytical results for array antennas with M-ary orthogonal modulation in Rayleigh fading were presented in [5], which used the analysis procedure given in [3]. No closed form expression for the BER was given in [5]. Here we consider the case of the more general Nakagami-m fading, which has been shown to provide a better fit to experimental data from a variety of fading environments including urban mobile radio and indoor radio propagation channels [4], [6]. We follow the analysis technique in [1] and develop a simple model, with a closed form expression for the BER, which can be used to calculate the system performance with array antennas operating in a Nakagami-m fading environment.

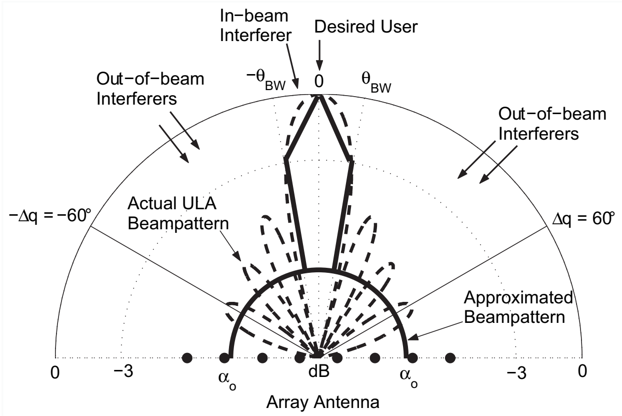

Fig. 1. Illustration of the beampattern approximation and partitioning of interferers.

The method proposed in [1] for the case of Uniform Linear Array (ULA) antennas is first briefly reviewed. In the procedure, it is assumed that the interferers are uniformly distributed in the coverage area comprising an angular sector of $120°$. The interferers are partitioned into two spatial equivalence classes − in-beam and out-of-beam − based on whether their Angles of Arrivals lie inside or outside the beam formed toward the desired user. A piece-wise beampattern is used to approximate the actual ULA beampattern, as illustrated in Fig. 1. In the pass band (or in-beam) the beampattern is modelled by a triangular function while in the stop-band (or out-of-beam) the beampattern is approximated by a constant attenuation factor representing an average side-lobe level. Because of the piece-wise approximation, the energy of each in-beam interferer is a random variable uniformly distributed within $[1/2, 1]$ (half power beamwidth region). Hence a correction factor of $3/4$ (average value) is applied for in-beam interferers. Using the above assumptions, the procedure proposes to adapt the single antenna BER results to array antenna systems by manipulating the terms accounting for noise and interference in the error probability formulas for single antenna receivers [1].

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II. System Model

We consider a CDMA system with K active users in the uplink. It is assumed that each of them uses M-ary orthogonal modulation and that the receiver is equipped with a Uniform Linear Array (ULA) antenna with inter-element distance d. The signal transmitted by the kth user is given by

$$s_k(t) = W_k^{(p)}(t)[a_k^{(I)}(t)\cos(\omega_c t) + a_k^{(Q)}(t)\sin(\omega_c t)] \quad (1)$$

where $W_k^{(p)}(t)$ is a Hadamard-Walsh function of dimension M which represents the pth orthogonal signal of the kth user, $p = 1, 2, \ldots, M$, and $a_k^{(I)}(t)$ and $a_k^{(Q)}(t)$ are the in-phase and quadrature channel PN random sequences respectively.

The kth user's signal propagates through a multipath fading channel with Angle of Arrival (AOA) θ_k. The channel impulse response between the kth transmitting user and the nth element of the array antenna is given by

$$h_{n,k} = \sum_{l=1}^{L} \beta_{k,l}\, e^{-j(\phi_{k,l}+2\pi\frac{d}{\lambda}(n-1)\sin\theta_k)}\delta(t-\tau_{k,l}) \quad (2)$$

where $\beta_{k,l}$, $\phi_{k,l}$ and $\tau_{k,l}$ are the path gain, phase and delay respectively and λ is the wavelength. The path gains are assumed to follow the Nakagami-m distribution with parameters $(m, \Omega_{k,l} = E[(\beta_{k,l})^2])$, where $E[\cdot]$ denotes the expectation. The Nakagami-m distribution includes Rayleigh distribution as a special case for $m = 1$ and can also accurately approximate Rician fading when $m > 1$ [6].

Under the above assumptions, the total received signal at the nth antenna is given by

$$\begin{aligned} x_n(t) \;=\; & \sum_{k=1}^{K}\sum_{l=1}^{L} \Big[\beta_{k,l}\, W_k^{(h)}(t-\tau_{k,l})\, a_k^{(I)}(t-\tau_{k,l}) \\ & \cos(\omega_c t + \varphi_{n,k,l}) + \beta_{k,l}\, W_k^{(h)}(t-\tau_{k,l}) \\ & a_k^{(Q)}(t-\tau_{k,l})\sin(\omega_c t + \varphi_{n,k,l})\Big] + \eta_n(t) \quad (3) \end{aligned}$$

where $\varphi_{n,k,l}$ is the overall phase which includes the path phase and the difference in propagation delays between the antennas and $\eta(t)$ is the Additive White Gaussian Noise (AWGN).

The receiver consists of a two-dimensional (2-D) RAKE receiver with a conventional Maximum Signal to Noise Ratio beamformer [7] followed by an L finger noncoherent RAKE combiner. The output of the RAKE combiner, which contains the desired user, noise, MAI and self interference components, is then used to estimate the transmitted data.

For the case of a single antenna receiver, the variances of the noise, MAI and self interference components are well known and are given by [4]

$$\sigma_N^2 = \frac{N_o}{2} \quad (4)$$

$$\sigma_M^2 = \frac{E_s}{3N_c}\sum_{k=1}^{K-1}\sum_{l=1}^{L} E[(\beta_{k,l})^2] \quad (5)$$

$$\sigma_I^2 = \frac{E_s}{3N_c}\sum_{l=1}^{L-1} E[(\beta_{k,l})^2] \quad (6)$$

where N_c is the spreading gain, E_s is the symbol energy and N_o is the noise power spectral density.

III. Performance Analysis

We apply the BER approximation procedure proposed in [1] to the case of noncoherent M-ary orthogonal modulation.

Let the modified variances of the noise, self interference and MAI be denoted as $\bar{\sigma}_N^2$, $\bar{\sigma}_I^2$ and $\bar{\sigma}_M^2$ respectively. Also let κ denote the number of in-beam interferers. Then number of out-of-beam interferers $= K-\kappa-1$. Applying the partitioning of the interferers, we get

$$\bar{\sigma}_M^2 = \frac{E_s}{3N_c}\left[f\sum_{k=1}^{\kappa}\sum_{l=1}^{L}E[(\beta_{k,l})^2] + \alpha_o\sum_{k=\kappa+1}^{K-1}\sum_{l=1}^{L}E[(\beta_{k,l})^2]\right] \quad (7)$$

where α_o is the attenuation factor for out-of-beam interferers and $f = 3/4$ is a correction factor for in-beam interferers.

The self interference due to the desired user's multipaths cannot be reduced by beamforming as the spatial channel in (2) is based on one AOA θ_k. Hence we have $\bar{\sigma}_I^2 = \sigma_I^2$.

In comparison with the power of the desired signal, the noise power at the output of the antenna array is reduced by N times, where N is the number of antenna elements [8]. Hence we get

$$\bar{\sigma}_N^2 = \frac{\sigma_N^2}{N} = \frac{\frac{N_o}{2}}{N} \quad (8)$$

The modified SINR at the output of the 2-D RAKE receiver is thus given by $\rho = \frac{E_s}{2\,\bar{\sigma}_T^2}$, where $\bar{\sigma}_T^2 = \bar{\sigma}_N^2 + \bar{\sigma}_I^2 + \bar{\sigma}_M^2$ is the total variance. For simplicity, we consider a uniform power delay profile i.e. $\Omega_{k,1} = \Omega_{k,2} = \cdots = \Omega_{k,L} = \Omega/L$. Substituting the values and and simplifying, we get

$$\rho = \frac{\gamma}{\frac{1}{N} + \frac{2\,\gamma}{3N_c}\left[(L-1) + \alpha_o\,(K-\kappa-1)\,L + f\,\kappa\,L\right]} \quad (9)$$

where average E_b/N_o of each multipath is $\gamma = \frac{\Omega}{L}\frac{E_s}{N_o} = \log_2(M)\frac{\Omega}{L}\frac{E_b}{N_o}$.

The average bit error probability for a conventional one-dimensional (1-D) RAKE receiver (i.e. single antenna without beamforming) in Nakagami fading over $L-$fold multipath diversity with Equal Gain Combining (EGC) is given by [4]

$$\begin{aligned} P_b^{(\text{1-D})} \;=\; & \frac{M/2}{M-1}\,\frac{1}{\Gamma(mL)}\sum_{n=1}^{M-1}(-1)^{n+1}\binom{M-1}{n} \\ \times\; & \left(\frac{m}{\frac{n}{n+1}\,\rho + m}\right)^{mL}\sum_{l=0}^{n(L-1)}\mathcal{B}_{ln}\,\frac{\Gamma(L+l)}{(n+1)^{L+l}} \\ \times\; & \sum_{i=0}^{l}\binom{l}{i}\frac{\Gamma(mL+i)}{\Gamma(L+i)}\left(\frac{\rho}{n(\rho+m)+m}\right)^{i} \quad (10) \end{aligned}$$

where m is the Nakagami fading parameter, $\Gamma(\cdot)$ is the gamma function and $\mathcal{B}_{ln}$ is the set of coefficients which can be computed recursively as [4]

$$\mathcal{B}_{ln} = \sum_{i=l-(L-1)}^{l}\mathcal{B}_{i(n-1)}\,\frac{1}{(l-i)!}\,I_{[0,(n-1)(L-1)]}(i) \quad (11)$$

where $\mathcal{B}_{00} = \mathcal{B}_{0n} = 1$, $\mathcal{B}_{l1} = \frac{1}{l!}$, $\mathcal{B}_{1n} = n$ and $I_{ab}(i)$ is the indicator function given by

$$I_{[a,b]}(i) = \begin{cases} 1, & a \le i \le b \quad (12a) \\ 0, & \text{otherwise.} \quad (12b) \end{cases}$$

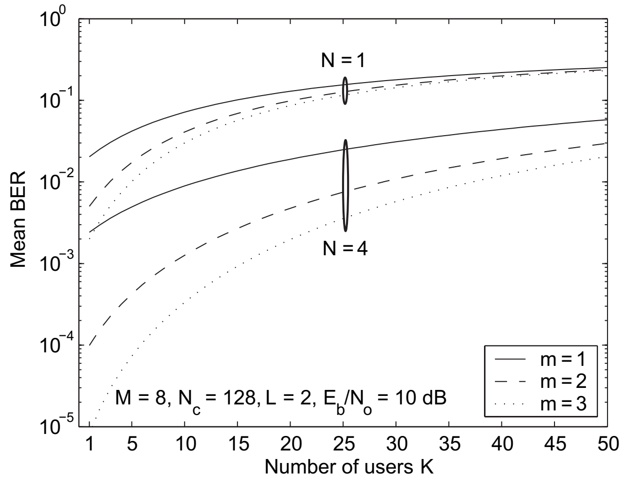

Fig. 2. Mean BER vs. Number of users K for fixed $E_b/N_o = 10$ dB, $M = 8$, $N_c = 128$, $L = 2$ paths/user, assuming $N = 1, 4$ antennas and $m = 1, 2, 3$ respectively (lines: analytical model).

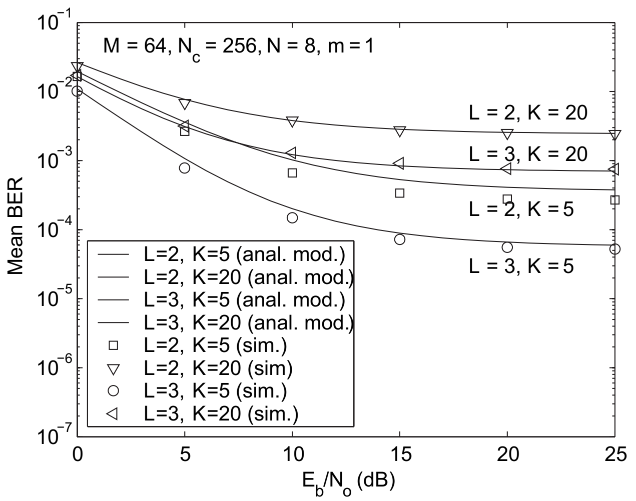

Fig. 3. Mean BER vs. E_b/N_o for $N = 8$ antennas, $N_c = 256$, $M = 64$, $m = 1$, assuming $K = 5, 20$ users and $L = 2, 3$ paths/user respectively (lines: analytical model, markers: simulations).

Using (10) and (9) and assuming a uniform distribution of interferers in a 120° angular sector, we obtain the average bit error probability of the 2-D RAKE receiver as [1]

$$P_b^{(2\text{-D})} = \sum_{\kappa=0}^{K-1} \chi\, \eta^{\kappa} \binom{K-1}{\kappa} P_b^{(1\text{-D})} \tag{13}$$

where $P_b^{(1\text{-D})}$ is the probability of bit error for the single antenna receiver given by (10) with ρ is given by (9), $\eta = \frac{2\,\theta_{BW}}{\Delta\theta}$ is the probability of an in-beam interferer, $\chi = (1-\eta)^{K-\kappa-1}$, $\Delta\theta = 120^\circ$ is the total coverage angle of the sector and $2\theta_{BW}$ is the total beamwidth towards the desired user. The optimized values of equivalent beamforming parameters $2\theta_{BW}$ and α_o used in this work are given in [1].

IV. Numerical Results

The analytical model described in the last section ((13) together with (10) and (9)) can be readily evaluated using a mathematical software package such as Matlab. Fig. 2 shows the mean BER versus the number of users K at fixed $E_b/N_o = 10$ dB, $M = 8$, processing gain $N_c = 128$ and $L = 2$ paths/user for $m = 1, 2, 3$ and $N = 1, 4$ antenna elements respectively. The reference curves in this figure are for the case of $N = 1$ single antenna receiver. It can be seen that a substantial performance gain is achieved by use of array antennas.

We also perform Monte Carlo simulations based on the system model presented in Section II to confirm the analytical results. For this purpose, we consider the case of $M = 64$ and $N_c = 256$ which is relevant to the uncoded IS-95 CDMA and cdma2000 systems. Fig. 3 shows the mean BER versus E_b/N_o (dB) with $N = 8$ element ULA, $K = 5, 20$ users and $L = 2, 3$ paths/user respectively for $m = 1$ i.e. (Rayleigh fading). It can be seen that for $K = 5$ users the simulation results (markers) are close to the analytical results (lines). However for $K = 20$ users, which represents a more realistic mobile communications scenario, the simulation results provide an even better agreement with the results obtained by the analytical model described in this letter.

V. Conclusions

In this letter, the performance of adaptive array antennas for CDMA with noncoherent M-ary orthogonal modulation in a Nakagami fading environment has been analysed. Using the approach presented in [1], an expression of the SINR at the output of the 2-D RAKE receiver has been derived which permits the BER to be readily evaluated using a closed form expression. The results obtained using this analytical model show good agreement with the (computationally intensive) simulation results.

References

[1] U. Spagnolini, "A simplified model to evaluate the probability of error in DS-CDMA systems with adaptive antenna arrays," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 578–587, Mar. 2004.
[2] V. K. Garg, *IS-95 CDMA and Cdma2000, Cellular/PCS System Implementation*. Eglewood Cliffs, NJ: Prentice Hall PTR, 2000.
[3] L. M. Jalloul and J. M. Holtzman, "Performance analysis of DS/CDMA with noncoherent M-ary orthogonal modulation in multipath fading channels," *IEEE J. Select. Areas Commun.*, vol. 12, pp. 862–870, June 1994.
[4] V. Aalo, O. Ugweje, and R. Sudhakar, "Performance analysis of a DS/CDMA system with noncoherent M-ary orthogonal modulation in Nakagami fading," *IEEE Trans. Veh. Technol.*, vol. 47, pp. 20–29, Feb. 1998.
[5] A. F. Naguib and A. Paulraj, "Performance of wireless CDMA with M-ary orthogonal modulation and cell site antenna arrays," *IEEE J. Select. Areas Commun.*, vol. 14, pp. 1770–1783, Dec. 1996.
[6] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*. New York: John Wiley & Sons, 2000.
[7] J. E. Hudson, *Adaptive Array Principles*. Peter Peregrinus Ltd., 1981.
[8] S. Choi and D. Yun, "Design of adaptive antenna array for tracking the source of maximum power and its application to CDMA mobile communications," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 1393–1404, Sept. 1997.