A Sliding Mode Approach to Robust Generation on dc-to-dc Nonlinear Converters.

ENRIC FOSSAS and DOMINGO BIEL

Abstract— **In this communication, a new approach is proposed for the design of a sliding feedback control scheme regulating nonlinear converters in AC signal generating task. The approach is based on the specification of a switching control law which accomplishes indirect asymptotic sinusoidal output voltage generation for the converters. The control scheme is found to be partially robust to perturbations, namely variations on the input voltage or/and in the load.**

Keywords— **Sliding Control Mode, Nonlinear Converters, Generation, Robustness.**

I. Introduction

This communication is concerned with nonlinear power converters and variable structure systems. We apply techniques of sliding control to nonlinear power converters in order to generate periodic output voltages. The basic nonlinear converters we deal with are the boost and the buck-boost. It is assumed that they are controlled in sliding mode by a time-periodic switching surface, which will be rewritten as an autonomous one. Time-periodic switching surfaces seems to be necessary if we are interested in a global treatment instead of a local one and, by this way, being able to derive a closed curve as the autonomous sliding surface.

We will have gone from tracking to generation; the inverse function theorem becames a basic tool in this way. The inverse function theorem presumes a condition on the derivative (it must be different from zero), in this case it is reflected in the design of the control function. It is not enough to take into account the sliding surface, the curve of equilibrium points have to be also considered. Moreover, it could be done a sliding motion on it, implying a not desired behaviour.

Another requirement for our system is to be robust in front of variations of the load and/or perturbations in the input voltage. Making use of the form of the control vector field, variations on the load can be considered as perturbations in the input voltage and to be counteracted by extern injections of voltage.

The reader is referred to [6] for elementary definitions and concepts inherent to sliding regimes theory used throughout this article, it is referred to [5] for the application of sliding control mode to the power converters. Papers closely related with this are [4] and [1] that will be presented in the CDC'96 conference.

This communication is organized as follows, in section 2 the elementary nonlinear power converters are presented as well as the hypotheses we assume throughout the paper. In section 3 the autonomous sliding surface and the switching control law are derived. Section 4 is devoted to make the control system robust. Finally, computer simulations are shown in section 5.

II. Nonlinear power converters

DC-to-DC switching power converters constitute a natural field of application of the sliding mode theory according to the abrupt topological changes that the circuit commanded by a discontinuous control action undergoes.

The nonlinear basic power converters boost and buck-boost can be described by the dynamical system

$$\begin{aligned}\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} &= \begin{pmatrix}0 & -\omega_0\\ \omega_0 & -\omega_1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}+\begin{pmatrix}\delta\\ 0\end{pmatrix}+ \\ &+\left[\begin{pmatrix}0 & \omega_0\\ -\omega_0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}+\begin{pmatrix}\gamma\\ 0\end{pmatrix}\right]u\end{aligned} \quad (1)$$

The values of the parameters appearing in the former equation for the nonlinear converters boost and buck-boost are

- boost converter. $\gamma = 0$, $\delta = b$,
- buck-boost converter. $\gamma = b$, $\delta = 0$,

where, $\omega_0 = \frac{1}{\sqrt{LC}}$, $\omega_1 = \frac{1}{RC}$ and $b = \frac{E}{\sqrt{L}}$.

The elimination of the control u in (1) yields a differential equation that relates the dynamics of the state variables. When the goal of having x_2 follow a given signal x_{2d} is reached, the variable x_1, proportional to the input current, satisfies an Abel's equation of the second kind:

$$x_1(b-\dot{x}_1) = \dot{x}_{2d}\left(x_{2d}+\frac{\gamma}{\omega_0}\right)+\omega_1 x_{2d}^2+\frac{\gamma}{\omega_0}\omega_1 x_{2d} \quad (2)$$

which, in general, is analytically unsolvable.

An exact treatment of the tracking problem with a sliding surface $S(x,t) := x_1 - k(t)$ involves $k(t)$ being a solution of (2). Since numeric simulations for usual testing functions x_{2d} show the domination of the bx_1 term, x_1 appears as a straight line with slope b that grows indefinitely. Thus the input current tends to infinite and an impossible physical implementation occurs. Hence, not bounded x_1 time-depending sliding surfaces cannot offer a satisfactory solution to the tracking output voltage problem in dc-to-dc nonlinear basic converters.

Nevertheless, a suitable change of variable allows the writing of (2) as a polinomyal differential equation that

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E. Fossas is with the Dpt. de Matemàtica Aplicada i Telemàtica. Universitat Politècnica de Catalunya. Mòdul C3. Campus Nord. Gran Capitán s/n. 08034 BARCELONA (SPAIN)
E-mail: fossas@mat.upc.es

D. Biel is with the Dpt. d'Enginyeria Electrònica. Universitat Politècnica de Catalunya. Escola Universitària Politècnica. c/ Víctor Balaguer s/n. 08800 VILANOVA I LA GELTRÚ (SPAIN).
E-mail: biel@eel.upc.es

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is found to have a maximum of one periodic solution [3]. It was also observed that, adjusting the initital conditions for a particular x_{2d}, solutions were found with an initially periodic aspect before the beginning of the raise. These two aspects lead to the conjecture that, assuming x_{2d} to be periodic, there is a periodic solution $k(t)$ of (2), that can be approximated with a truncated Fourier developpment $k_a(t)$.

Finally, a sliding control mode based on the time-dependent sliding surface $S(x,t) := x_1 - k(t) = 0$, ($S(x,t) := x_1 - k_a(t) = 0$ in practice) can be designed. It can be proved with some additional hypotheses on x_{2d} in the buck-boost case, that in the ideal sliding dynamics, the error $|x_2 - x_{2d}|$ tends asymptotically to zero.

Summarizing, the assumption we presume from now on is

The control policy defined by the surface

$$S(x,t) := x_1 - k(t) = 0 \tag{3}$$

and the control law

$$u(x,t) = \begin{cases} 0 & \textit{if } (\omega_0 x_2 + \gamma) S(x,t) > 0 \\ 1 & \textit{if } (\omega_0 x_2 + \gamma) S(x,t) < 0 \end{cases} \tag{4}$$

provides us with a sliding motion on the surface $S(x,t) = 0$, and in ideal sliding dynamics the error $|x_2 - x_{2d}|$ tends asymptoticaly to zero

Note that zero should be substituted by ε when $k_a(t)$ is used instead of $k(t)$.

III. Generating Signals

If $\dot{x}_{2d}(t) \neq 0$, by the inverse function theorem $t = t(x_2)$ and the former time-dependent sliding surface can be written as $S(x, t(x_2)) := x_1 - k(x_2) = 0$ beeing autonomous. The discontinuous control policy which, with the switching surface (3), provides a time-dependent control scheme, can also be expressed as a function on x_1, x_2. This autonomous control policy steers the system to the switching surface, providing a sliding motion on it and the ideal sliding dynamics arranged in advance.

Actually, the ideal sliding dynamics will be parametrized as

$$x_1 = \phi_1(t) \qquad x_2 = \phi_2(t)$$

Then the inverse function theorem can be used everywhere except when $\dot{x}_1(t) = 0$ and $\dot{x}_2(t) = 0$ that is, it cannot be used on the curve of equilibrium points $C(x_1, x_2) = 0$. In fact, the control policy depends also on the sign of the function $C(x_1, x_2)$.

Applying the former procedure to a nonlinear converter from which we wont an output voltage as

$$x_{2d} = A \sin(\omega t) + B$$

and approximating the periodic solution $k(t)$ of (2) by a truncated Fourier expansion $k_a(t)$, we have

- $C(x_1, x_2) = \left(-\omega_0 x_1 b + \omega_1 \gamma x_2 + \omega_0 \omega_1 x_2^2\right) = 0$
- $S(x_1, x_2) :=$
  $= \left[x_1 - a_0 - b_1\left(\frac{x_2 - B}{A}\right) - a_2\left(1 - 2\left(\frac{x_2 - B}{A}\right)^2\right)\right]^2 -$
  $- \left(a_1 + 2b_2\left(\frac{x_2 - B}{A}\right)\right)^2\left(1 - \left(\frac{x_2 - B}{A}\right)^2\right) = 0$
- $u(x_1, x_2) =$
  $= \begin{cases} 0 & \text{if } (\omega_0 x_2 + \gamma) C(x_1, x_2) S(x_1, x_2) > 0 \\ 1 & \text{if } (\omega_0 x_2 + \gamma) C(x_1, x_2) S(x_1, x_2) < 0 \end{cases}$

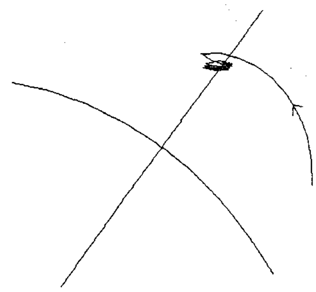

Fig. 1. Undesired behaviour in $C(x_1, x_2) \cap S(x_1, x_2)$

As in [2], there can be found conditions on the parameters A, B and ω in order to a sliding motion exists. However it should be mentioned too that the study made in the time-dependent case suffices; on the other hand, an analytical study cannot be done because the exact solution of equation (2) has had to be approximated by $k_a(t)$.

Simulations are in perfect agreement with this results. A sliding motion can be observed on the designed surfaces.

Remark that the equation of the curve of equilibrium points $C(x_1, x_2)$ appears in the definition of the control as a factor in the decision of changing values for $u(x_1, x_2)$. It is the cause of a not desired phenomenon that can be observed in a neigborhood of the intersection between $C(x_1, x_2)$ and $S(x_1, x_2)$. Namely, depending on how trajectories cut the curve $C(x_1, x_2)$ a sliding motion appear on it. The problem has been solved by keeping the control value for some consecutives sample periods.

In figure 1 there is sketched this phenomenon, its effect on the output voltage gets a sudden lost of signal generation ending with an equilibrium point, as it can be seen in figure 2.

IV. Robustness

This section shows how we make our system robust in front of variations on the load. It is assumed that the input voltage can be modified by the addition of a voltage compensator that will counteract the variation on the load.

It is well known that the sliding control, in fact the ideal sliding dynamics, is robust in front of perturbations included in the input channel distribution Δ_u. Hence, two perturbations can be considered equivalent if their difference belongs to Δ_u. In the light of that, variations on the load are equivalent to perturbations in the input voltage,

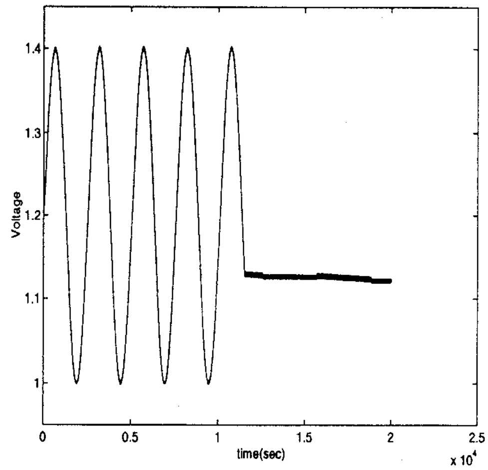

Fig. 2. Normalized output voltage. Undesired behaviour.

which will be appropriatelly counteracted.

Namely, from equation 1 and the former comments, perturbation vector fields for the basic nonlinear converters can be described as

$$p(x_1, x_2) = (\hat{\delta} + \hat{\gamma}u, -\hat{\omega}_1 x_2)$$

where $\delta = \delta_{nom} + \hat{\delta}$, $\gamma = \gamma_{nom} + \hat{\gamma}$ and $\omega_1 = \omega_{1,nom} + \hat{\omega}$ in the perturbed system.

On the other hand, the input channel distribution is generated by the vector field

$$g(x_1, x_2) = (\omega_0 x_2 + \gamma, -\omega_0 x_1)$$

thus the perturbation vector field $p(x_1, x_2)$ is equivalent to

$$p'(x_1, x_2) = \left(-\frac{\hat{\omega}_1 x_2}{\omega_0 x_1}(\omega_0 x_2 + \gamma) + (\hat{\delta} + \hat{\gamma}u), 0 \right)$$

which, in turn is counteracted by substracting the compensator

$$(\hat{\delta} + \hat{\gamma}u) - \frac{\hat{\omega}_1 x_2}{\omega_0 x_1}(\omega_0 x_2 + \gamma)$$

from the input signal

$$\delta + \gamma u$$

As it has been deduced by the reader, we also assume that the value of the parameters are known, as well as the load variations and the input voltage perturbations.

V. Examples

A. Boost converter

The following simulations correspond to a boost converter with nominal parameters $\omega_0 = 502.52$, $\omega_1 = 454.55$, $b = 372.67$, and the desired signal $x_{2d} = 1.2 + 0.2\sin(500t)$ which correspond to $L = 18mH.$, $C = 220\mu F.$, $R = 10\Omega.$ and $E = 50V.$.

Figure 3 shows a trajectory which evolves on the sliding surface once the system is controlled by the former described switching law.

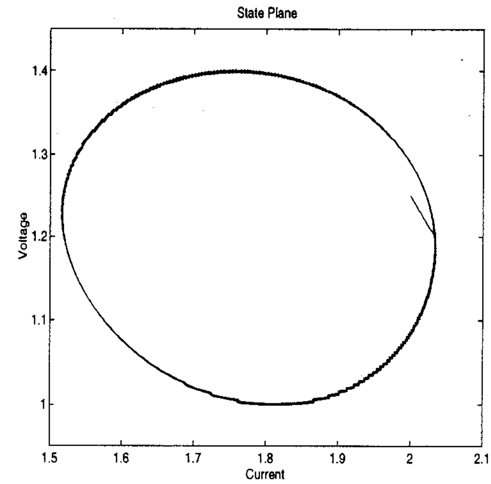

Fig. 3. Phase state diagram. Controlled dynamics

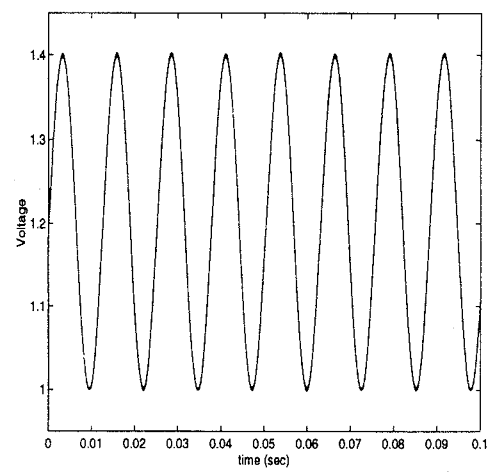

Fig. 4. Normalized output voltage under load variation

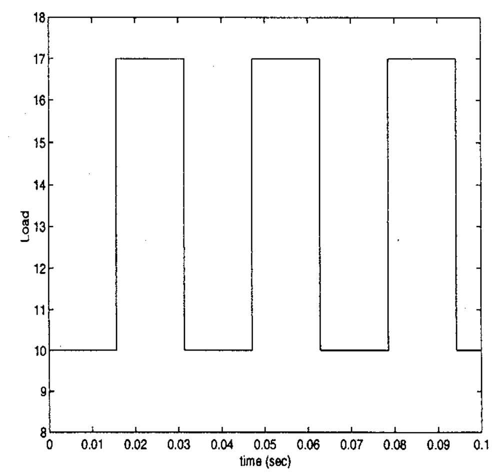

Fig. 5. Load variation

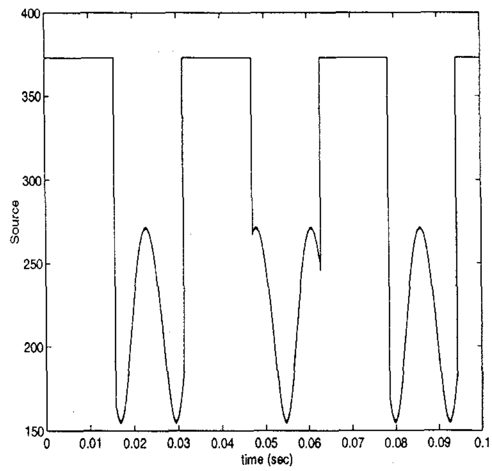

Fig. 6. Voltage compensator counteracting the load variations

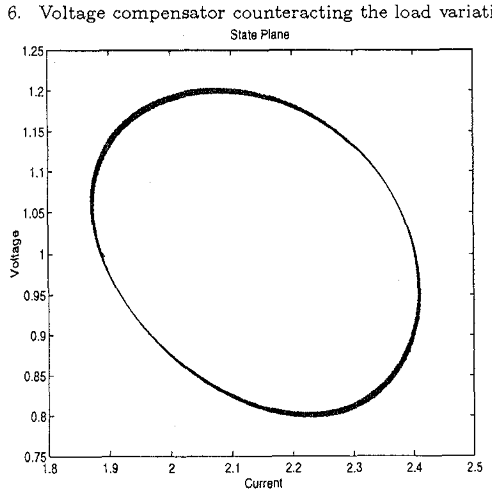

Fig. 7. Phase state diagram. Controlled dynamics under load variations

Figure 4 shows the output voltage in front of a load variation, the load is assumed to be a pulsed signal of height 7 and initial condition 10Ω, it is shown in figure 5. Figure 6 illustrates the source to be used in order to compensate the load perturbations.

B. Buck-boost converter

The following simulations correspond to a buck-boost converter with nominal parameters $\omega_0 = 502.52$, $\omega_1 = 454.55$, $b = 372.67$, and the desired signal $x_{2d} = 1 + 0.2\sin(500t)$ which correspond to $L = 18mH.$, $C = 220\mu F.$, $R = 10\Omega.$ and $E = 50V.$.

Figure 7 shows a trajectory which evolves on the sliding surface in front of load variations, once the system is controlled by the former described switching law.

Figures 8 and 9 show the input current and the output voltage, the load is assumed to be a pulsed signal of height 10 and initial condition 8Ω. Figure 10 illustrates the

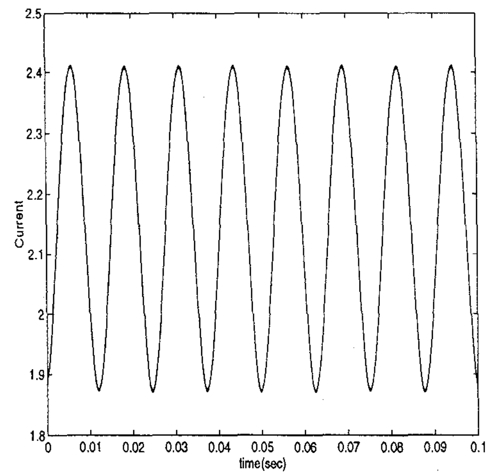

Fig. 8. Normalized input current under load variation

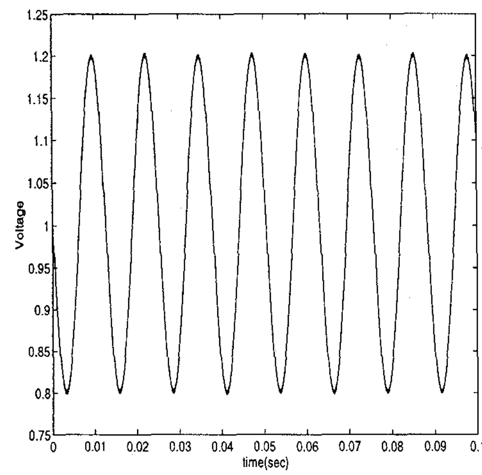

Fig. 9. Normalized output voltage under load variation

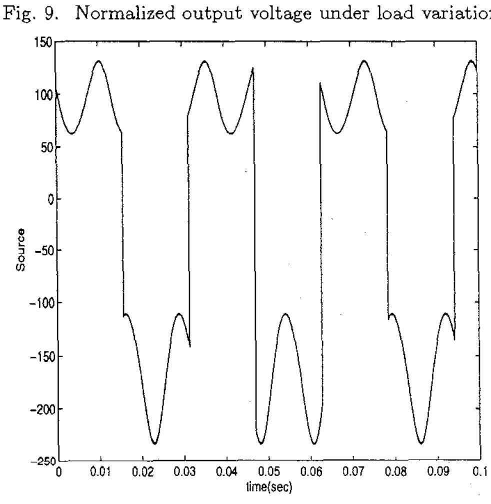

Fig. 10. Voltage compensator counteracting the load variations

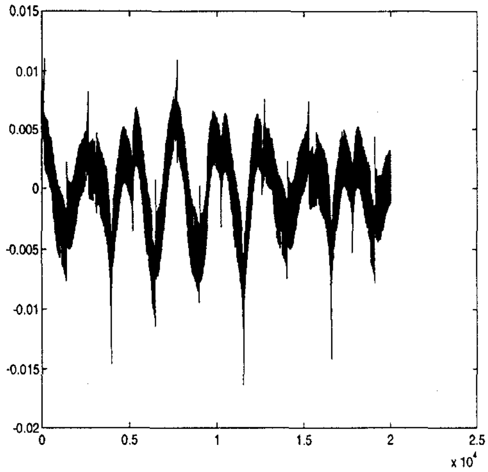

Fig. 11. Voltage compensator counteracting the load variations

source to be used in order to compensate the load perturbations. The pulsating error; that is, the relative ω's error $\frac{\omega(obtained)-\omega}{\omega} \sim 0.003$. The error $|x_2 - x_{2d}|$ can be seen in figure 11. Note that it remains less than 0.015 in spite of the load variations.

VI. Conclusions

In this communication it has been proposed an autonomous sliding feedback control scheme regulating nonlinear converters in AC signal generating task. The autonomous sliding surface is derived in the light of the implicit function theorem from a time-dependent one that is assumed to exist fulfilling a sliding motion.

This procedure is specified for nonlinear basic converters. It has been observed an undesired behaviour that can appear in a neigborhood of the intersection between the curve of equilibrium points and the sliding surface. It has been proposed too a design to avoid this behaviour. The control scheme is found to be robust in front of perturbations, namely variations in the load.

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