Barkhausen Criterion and Another Necessary Condition for Steady State Oscillations Existence

Igor Balaz*, Zdenko Brezovic*, Marian Minarik*, Vladimir Kudjak*
* Institute of Electronics and Photonics
Slovak University of Technology
Bratislava, Slovakia
vladimir.stofanik@savba.sk

Vladimír Stofanik*, +
+ Institute of Physics
Slovak Academy of Sciences
Bratislava, Slovakia
vladimir.stofanik@savba.sk

Abstract— **Recent times have given rise to interesting discussions about the necessary and sufficient criteria for steady state oscillations of electronic circuits. The aim of this paper is to point out that the Barkhausen criterion (Bc), while a necessary condition for oscillations, but not a sufficient one, could be supplemented with another necessary condition. Here, we would like to take account of a feature of phase vs. frequency characteristics of linear feedback networks for oscillators. We analyze Twin-T (DT) oscillator and modified Wien (MW) oscillator according to [6], [7], from the point of linear circuit theory by means of open loop characteristic equations and root-locus diagrams for some parameters values where derivatives of the phase vs. frequency open loop feedback systems can be positive or negative, (in the vicinity of the frequencies where the Bc is fulfilled). Characteristic equations of both oscillators and their root-locus diagrams as a function of gain of ideal linear amplifier (AMP) are compared and estimated. For nonlinear amplifier, nonlinear ordinary differential equations have been solved and compared with the assistance of MATLAB, MathCAD and MICRO-CAP (MC10). In addition, we present experimental investigation of the impact of phase vs. frequency characteristic properties on steady state oscillation existence in feedback oscillators in other article as well [15]. All obtained results have confirmed that feedback systems with positive derivatives of phase vs. frequency characteristics (in the vicinity of frequencies where Bc is fulfilled) are incapable of producing steady state oscillations. So we can say that the requirement for negative derivatives of phase vs. frequency characteristic of the feedback network could be understood as a necessary condition for oscillations existence. It is a novel look at the role of this characteristic for steady state oscillations existence.**

Keywords—Barkhausen criterion; oscillators; characteristic equation of feedback systems; root locus approach; nonlinear ordinary differential equations.

I. INTRODUCTION

For many decades we have seen that some textbooks of circuit theory have addressed the Bc as a necessary and sufficient criterion for the existence of stable periodic processes in electronic circuits. Other basic circuit theory textbooks introduced the Bc as a necessary condition for oscillations but said nothing about cases where the Bc is fulfilled in a circuit but oscillations are impossible (Clark & Hess [1], Northrop [2], Gonzales [3]).

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A discussion about the Bc in recent years is demonstrated by means of some variants of RC feedback networks, mostly second order and using various active elements and different approaches, as well [4], [5], [6], [8], [10], [12], [13], [14].

E. Lindberg investigated the classical Wien oscillator with nonlinear elements made from two antiparallel diodes (the OPAMP was linear and frequency independent) and classified oscillators on the basis of their topology [4].

A.S. Elwakil analyzed and explained the origin of latch-up in one modification of the classical Wien oscillator (WO). There it was shown how latch-up can be eliminated [5].

V. Singh [6] discussed the failure of the Bc on an example of an equivalent form of classical Wien oscillator (that was also a modification of the WO). He had an attempt to draw attention to the close relation between the Bc and Nyquist criterion.

L. von Wangenheim [7] recently commented on Singh's paper where two variants of the modified WO (MWO) have been analyzed and made there some serious remarks about steady state oscillation existence.

In [8] M. Taher Abuelma´ati presents a catalogue of RC oscillator circuits and shows that the use of the Bc for startup condition determination of oscillations yields inaccurate results. His conclusions were critically reviewed in [12].

A very radical and controversial refutation of the Bc has been formulated by K. Lundberg [10]. We will not comment on his exclamation “Down with the Barkhausen criterion” (because it sounds as a joke).

B. N. Biswas [13] pointed out the Bc cannot be considered as a full-proof criterion for oscillation in practical oscillators. For nonlinear analysis of a growth of oscillation he has applied the slowly varying function approach. He demonstrated stable and unstable limit cycles for soft and hard-self excitations by means of nonlinear active element transconductance.

M. H. W. Hoffmann [14] made an attempt to give a new criterion for safe start-up oscillation on one modification of the WO example analyzing its equilibrium points.

Beside all the above variants of RC feedback circuits, there are other RC and LC ones (with lumped or distributed parameters) having similar features from the viewpoint of their amplitude (module) and phase vs. frequency (argument) characteristics [11].

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II. Analysis and mutual comparison of DT and MW feedback systems

Consider the feedback system with DT-circuit and amplifier shown in Fig. 1. The amplifier (AMP) can be noninverting or inverting, ideal linear, ideal nonlinear one or real AMP, according to DT-circuit parameters and selection of analysis method.

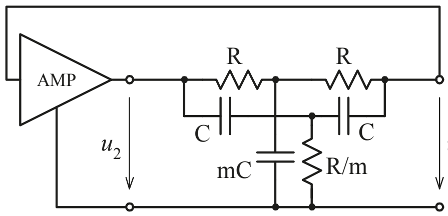

Figure 1. Equivalent form of DT oscillator.

In this section we will suppose AMP as an ideal and linear VCVS characterized by voltage amplification A. For linear analysis in Fig.1 we take a transfer function of the DT-circuit in normalized form

$$T(s)=\frac{u_1(s)}{u_2(s)}=\frac{s^2+(2/m-1)s+1}{s^2+(2/m+1+m)s+1} \qquad (1)$$

where $s = pRC$ or $s = j\Omega$, $\Omega = \omega RC$ and m is a positive number.

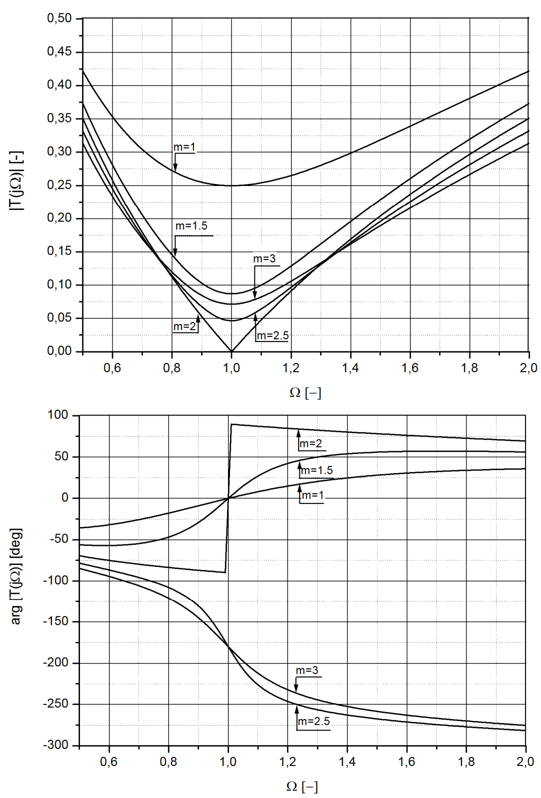

Figure 2. Amplitude vs. frequency characteristics (top) and phase vs. frequency characteristics (bottom) of the DT network for different values of the *m*.

Corresponding module and argument characteristics obtained from (1) for some values of *m* are depicted in Fig. 2. A modification of classical Wien oscillator is shown in Fig. 3 (redrawn here from [7]).

The transfer function of the modified feedback circuit for R_1=R_2 and C_1=C_2 has a normalized form

$$T(s)=\frac{u_1(s)}{u_2(s)}=\frac{s^2+2s+1}{s^2+3s+1} \qquad (2)$$

where *s* and Ω are the same as in (1).

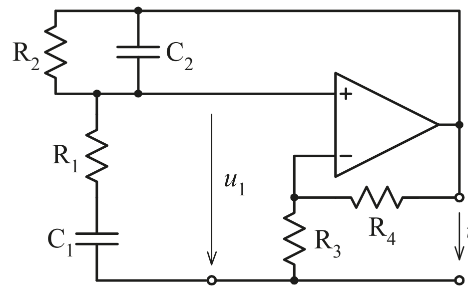

Figure 3. Modification of the Wien oscillator circuit.

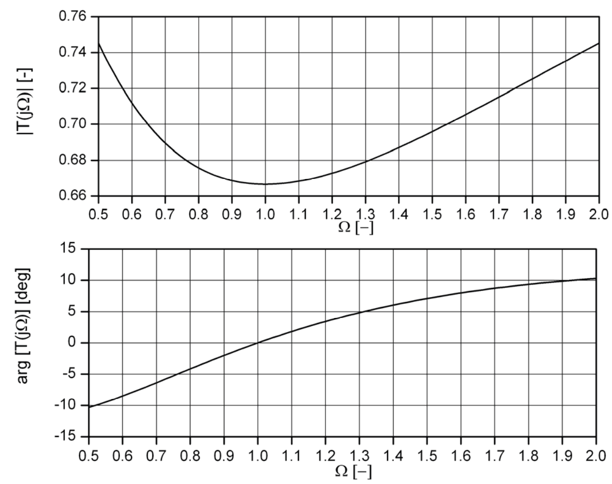

Figure 4. Amplitude vs. frequency characteristic (top) and phase vs. frequency characteristic (bottom) of modified Wien feedback circuit.

Comparing DT and modified Wien feedback network module characteristics we can see that the first one has the character of the notch filter (for *m*=2, $\Omega=\Omega_0=1$) where changing value of *m* the minimum will be changed, as well. For modified Wien circuit we have not a possibility to tune the module characteristic and its minimum (for $\Omega_0=1$) rests only on level 2/3 (Fig. 4).

Now, let us have a look at argument characteristics of both networks. In the case of DT feedback network their character in the vicinity $\Omega_0=1$ changes according to variation $2>m>2$ (Fig. 2). At the modified Wien feedback network (Fig.4) we can see a positive derivative of its argument characteristic in the whole range of frequencies.

Now let us pay attention to the properties of DT feedback network with AMP according to Fig. 1. When we will suppose the ideal and linear AMP which has voltage amplification *A*, then using (1) and Fig. 1, we can write the transfer function of the feedback system in obvious form

$$H(s) = \frac{A}{1 - A \cdot T(s)} \tag{3}$$

where $A \cdot T(s)$ is defined as the loop gain.

Since $T(s) = N(s)/D(s)$, the characteristic equation (CE) of this system can be written in the following form

$$D(s) - A \cdot N(s) = 0 \tag{4}$$

where

$$\begin{aligned} N(s) &= s^2 + (2/m - 1)s + 1 \ , \\ D(s) &= s^2 + (2/m + 1 + m)s + 1. \end{aligned} \tag{5}$$

Combining (4) and (5) we obtain the CE in a suitable form for analysis

$$s^2 + \frac{2 \cdot (1 - A)/m + 1 + m + A}{1 - A} s + 1 = 0 \tag{6}$$

Applying the root-locus technique to the solution of (6) and changing gain *A*, we obtain a plot for *m*>2 in Fig. 5.

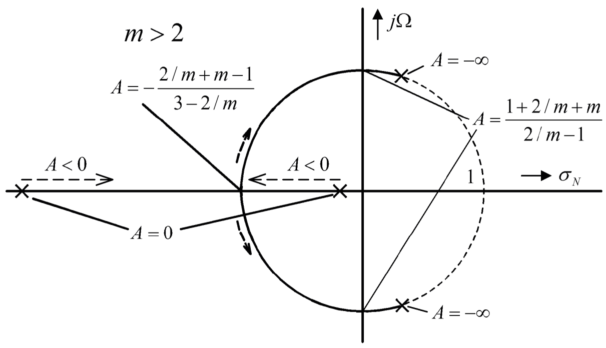

Figure 5. Root-locus diagram for DT oscillator (*m*>2).

From the root-locus diagram it is evident that by changing amplification (A<0) roots of the CE move together to the point (-1,0) and afterwards a pair of complex conjugate roots occurs and approximation to axis jΩ can be observed. This is a typical case of active filter creation with selective properties.

As we can see from Fig. 5 the minimal value of *A* for oscillation origin is given as follows

$$A = (2/m + m + 1)/(2/m - 1) \tag{7}$$

Decreasing value of *A* (*A* is negative) according to (7) the system will oscillate similarly as a classical Wien oscillator for *A*>3. The difference is only in inverting AMP for the first case and noninverting one for the second one.

Solving CE (6) for *m*=1 as a function of *A,* we get a root-locus diagram in Fig. 6. To be the Bc fulfilled, the feedback system need here positive values of *A*. For all values *m*<2 roots of CEs of feedback system give us similar plots as shown in Fig. 6. Here e.g. we can take *m*=1 and then obtain CE from (6) as follows

$$s^2 + \frac{4 - A}{1 - A} s + 1 = 0 \,\cdot \tag{8}$$

Modified WO (MWO) in the case of ideal and linear AMP has the CE in a similar form

$$s^2 + \frac{3 - 2A}{1 - A} s + 1 = 0 \,\cdot \tag{9}$$

The similarity of CE of (8) and (9) is not accidental, because their module and argument characteristics are similar as well.

At he interval of *A* <0, 1) two roots of the CE are real and lie in LHP. When *A*=1 one root goes to $\pm\infty$ and the second one passes across imaginary axis to RHP (that is true for both cases, DT and MWO).

For the interval of *A* (1, 2> the diagram has two roots on real axis σ_N at RHP and they move from the left and from the right side to the point “1” where comes to the coincidence of both roots (at *A*=2). For MWO this value *A*=1.25.

For *A*>2 (*A*>1.25, for MWO) doubled roots bifurcate and create a pair of complex conjugated roots, moving to the imaginary axis.

For *A*=4 (and *A*=1.5 for MWO) the complex conjugated roots lie on the imaginary axis,

For *A*>4 (and *A*>1.5 for MWO) these roots pass across the imaginary axis to LHP.

The Barkhausen criterion is fulfilled for both circuits, if CE roots reach axis $j\Omega$.

However, we must say that both feedback circuits (Fig. 1 for *m*<2 and Fig. 3) are incapable of generating steady state oscillations due to the fact that for the value *A*=1 one real root crosses axis $j\Omega$ from LHP to RHP and the second one from $-\infty$ to $+\infty$ and that it leads to a rise of relaxations or directly to a saturation of systems.

Another previously investigated RC and LC feedback circuits [11] have similar argument characteristics as were shown in Fig.2 (module ones as well). Root locus diagrams similar to those in Fig.6 are typical for all these cases where $d\varphi / d\omega$ (in the vicinity of quasi-resonant frequencies of modular characteristics) is positive. Due to this fact, steady state oscillations at such feedback systems cannot exist.

In [7] and discussion [9] where MW and DT oscillator were analyzed from various aspects, correct conclusions about non existence of continuous oscillations had been made, but the fact of the passing of roots from LHP to RHP for *A*>1 and a character of an argument characteristic has not been taken into account.

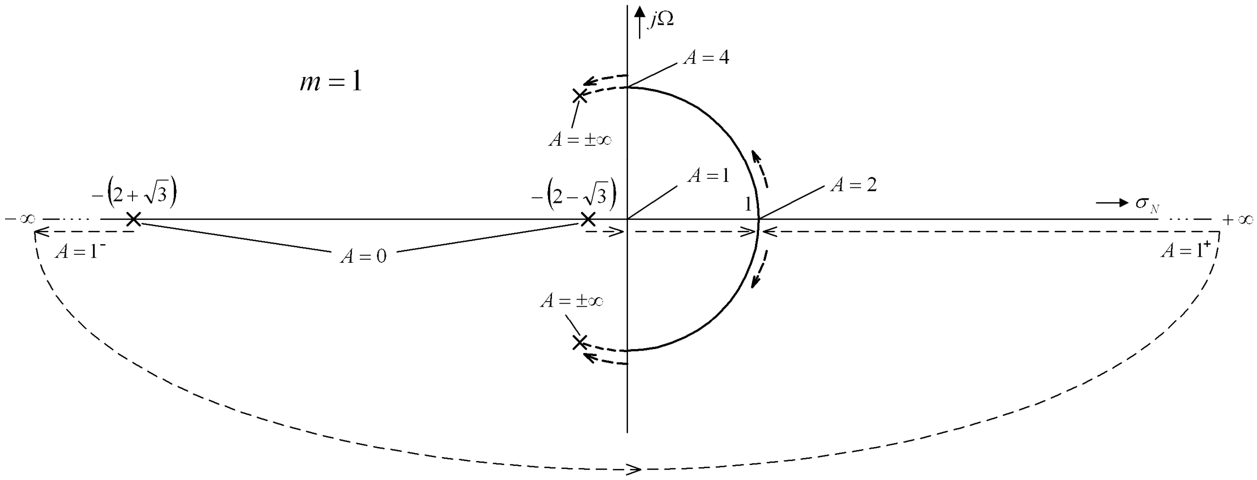

Figure 6. Root-locus diagram for DT oscillator (m=1).

III. NONLINEAR ANALYSIS OF DT AND MW OSCILLATORS

At computer simulations some approximations of a nonlinear transfer characteristic of an active element were considered and tested. Here we will consider an ideal nonlinear AMP with nonlinear transfer characteristic function in the form

$$u_2 = A \cdot \operatorname{atan}(u) \qquad (10)$$

We can write a differential equation of the DT oscillator, using (1) and (10), as

$$\ddot{u} = \frac{\dot{u}\left(\frac{aA}{1+u^2} - b\right) - u\left(\frac{2A\dot{u}^2}{(1+u^2)^2} + 1\right) + A \cdot \operatorname{atan}(u)}{1 - \frac{A}{1+u^2}} \qquad (11)$$

where

$$\ddot{u} = \frac{d^2u}{d\tau^2},\ \dot{u} = \frac{du}{d\tau},\ \tau = \frac{t}{RC},\ a = \frac{2}{m} - 1,\ b = \frac{2}{m} + m + 1 .$$

In this manner we can obtain a differential equation for the MW oscillator using (2) and (10)

$$\ddot{u} = \frac{\dot{u}\left(\frac{2A}{1+u^2} - 3\right) - u\left(\frac{2A\dot{u}^2}{(1+u^2)^2} + 1\right) + A \cdot \operatorname{atan}(u)}{1 - \frac{A}{1+u^2}} \qquad (12)$$

The behavior of the DT oscillator for $m<2$ is illustrated in following two figures (Fig. 7 and Fig. 8), where numerical solutions of (11) obtained with the assistance of programs MATLAB and MathCAD, as well as the results of simulation of the DT oscillator in program MC10 are compared. Figure 7 relates to DT oscillator with A=4.1, m=1; and Fig. 8 relates to DT oscillator with A=3.9, m=1. Identical initial conditions $u(0) = 0.05\mathrm{V}$, $\dot{u} = 0$ were considered in both cases.

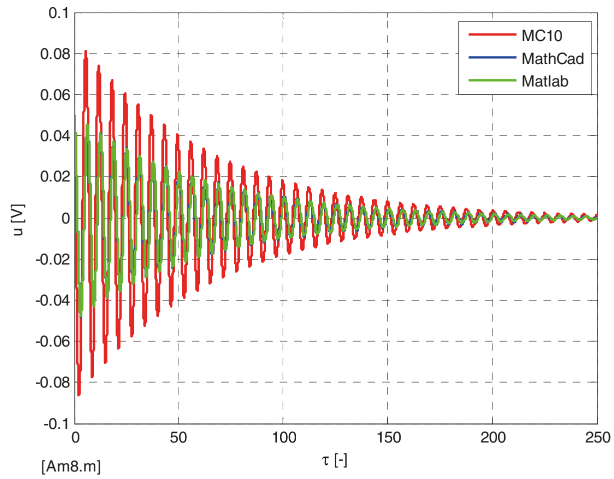

Figure 7. Solution of the differential equation (11) for m=1, A=4.1 in MATLAB, MathCAD and corresponding DT oscillator simulation in MC10.

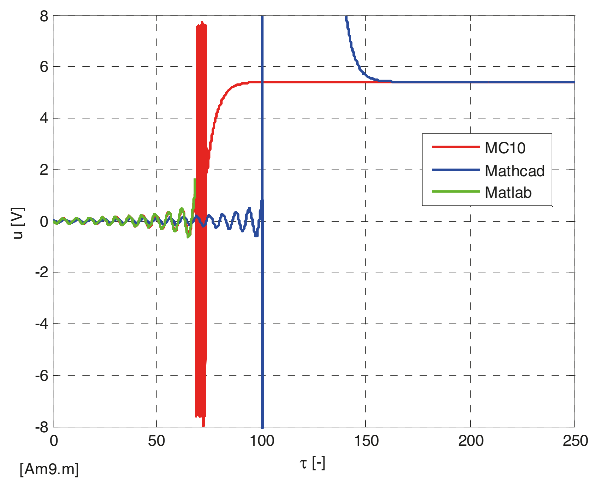

Figure 8. Solution of the differential equation (11) for m=1, A=3.9 in MATLAB, MathCAD and corresponding DT oscillator simulation in MC10.

The results presented in Fig. 7 are in accordance with circle root-locus diagram for DT oscillator (m=1) presented in Fig. 6. In this case a minimal required “initial” amplification of an active element for the origin of oscillation is 4 (Fig. 6) and for an amplification greater than the minimal value 4 (e.g. 4.1) steady state oscillations are not present.

The solutions (in the second example) presented in Fig. 8 are only partly identical. They are approximately equal from the beginning to a time at which the program MATLAB ends its operation because it is unable to meet integration tolerance. From this time to a certain time the rest solutions (in MathCAD and MC10) are ambiguous and finally they are practically the same again. This ambiguity implies a problem of discovering the investigated properties of the circuit in this way and calls for more sophisticated methods than are used in our case.

However the results in Fig. 8 showed that in spite of the originated oscillations which are in accordance with the circle root-locus diagram in Fig. 6, steady state oscillations, for nonlinearity of an active element expressed by function “atan”, are not present as well.

The behavior of the MW oscillator described by differential equation (12) is similar to one of the above DT oscillator for m<2 (in our case m=1). The solution of the differential equations (11) and (12) and corresponding simulations in MC10 are comparable. This fact can be illustrated in two examples (Fig. 9) of solutions of (12) with assistance of program MATLAB. The first solution relates to A=1.55 and the second one relates to A=1.45. The identical initial conditions $u(0)=0.05\mathrm{V}$, $\dot{u}=0$ were considered in both cases. The minimal amplification value of an active element for origin of oscillations in this case is A=3/2=1.5. In Fig. 9 we can see again that MATLAB ends its operation for the same reason as in the previous case; and again steady state oscillations are not present here.

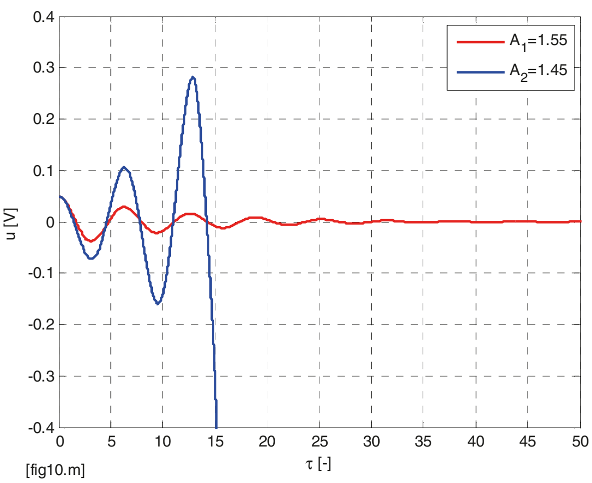

Figure 9. Solution of the differential equation (12) of the MW oscillator for A=1.55 and for A=1.45 in MATLAB.

IV. Conclusions

So the Barkhausen criterion is fulfilled (for A=1.5 in the case of MW oscillator and for A=4 in the case of DT oscillator if m=1) but these circuits cannot oscillate. This fact was confirmed not only by means of the linear analysis solving appropriated CEs and their root locus diagrams but nonlinear analysis and experiments as well [15].

Our investigation of these feedback circuits leads us to assume that the demand to

$$d\varphi / d\omega\big|_{\omega_0} < 0$$

could be taken as another necessary condition for steady state oscillation existence and understood as a supplement of the Barkhausen criterion.

Acknowledgment

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