

The following is a continuous non-stationary mathematical model describing the spread of noise with consideration of some of the major factors of the dissipative medium. The proposed model is based on the system of Maxwell's equations and the telegraph equations, given the fact, that the force of gravity and stratification of geophysical environments strongly modify sound waves propagating through them.

The system of equations with small parameters ε_i ($i=1,2$); initial conditions and boundary conditions of mixed type:

$$\left\{ \begin{array}{l} \frac{\partial^2 u(t, x)}{\partial t^2} + \varepsilon_1 \cdot \frac{\partial u(t, x)}{\partial t} = \Delta u(t, x) + \rho(t, x), \quad x \in D, \quad t \in [0, T], \\ \frac{\partial^2 \rho(t, x)}{\partial t^2} + \varepsilon_2 \cdot \frac{\partial \rho(t, x)}{\partial t} = \Delta \rho(t, x) + u(t, x) + F(t, x), \quad x \in D, \quad t \in [0, T], \\ u(t, x)|_{t=0+0} = u_0(x), \quad x \in \bar{D}, \\ \rho(t, x)|_{t=0+0} = \rho_0(x), \quad x \in \bar{D}, \\ u(t, x)|_{x_i=l_i^{(i)}+0} = u_1^{(i)}(t; x / \{x_i\}), \quad x / \{x_i\} \in \bar{D} / [l_1^{(i)}, l_2^{(i)}], \quad i = \overline{1,3}, \quad t \in [0, T], \\ u(t, x)|_{x_i=l_i^{(i)}-0} = u_2^{(i)}(t; x / \{x_i\}), \quad x / \{x_i\} \in \bar{D} / [l_1^{(i)}, l_2^{(i)}], \quad i = \overline{1,3}, \quad t \in [0, T], \\ \left. \frac{\partial \rho(t, x)}{\partial x_i} \right|_{x_i=l_1^{(i)}+0} = \rho_1^{(i)}(t; x / \{x_i\}), \quad x / \{x_i\} \in \bar{D} / [l_1^{(i)}, l_2^{(i)}], \quad i = \overline{1,3}, \quad t \in [0, T], \\ \left. \frac{\partial \rho(t, x)}{\partial x_i} \right|_{x_i=l_2^{(i)}-0} = \rho_2^{(i)}(t; x / \{x_i\}), \quad x / \{x_i\} \in \bar{D} / [l_1^{(i)}, l_2^{(i)}], \quad i = \overline{1,3}, \quad t \in [0, T], \end{array} \right. \quad (1)$$

Consistency conditions of initial and boundary functions:

$$\begin{aligned} u_1^{(1)}(t, l_1^{(2)} + 0, x_3) &= u_1^{(2)}(t, l_1^{(1)} + 0, x_3), \quad t \in [0, T], \quad x_3 \in [l_1^{(3)}, l_2^{(3)}]; \\ u_1^{(1)}(t, x_2, l_1^{(3)} + 0,) &= u_1^{(3)}(t, l_1^{(1)} + 0, x_2), \quad t \in [0, T], \quad x_2 \in [l_1^{(2)}, l_2^{(2)}]; \\ u_1^{(2)}(t, x_1, l_1^{(3)} + 0,) &= u_1^{(3)}(t, x_1, l_1^{(1)} + 0), \quad t \in [0, T], \quad x_1 \in [l_1^{(1)}, l_2^{(1)}]; \\ u_2^{(1)}(t, l_2^{(2)} - 0, x_3) &= u_2^{(2)}(t, l_2^{(1)} - 0, x_3), \quad t \in [0, T], \quad x_3 \in [l_1^{(3)}, l_2^{(3)}]; \\ u_2^{(1)}(t, x_2, l_2^{(3)} - 0,) &= u_2^{(3)}(t, l_2^{(1)} - 0, x_2), \quad t \in [0, T], \quad x_2 \in [l_1^{(2)}, l_2^{(2)}]; \\ u_2^{(2)}(t, x_1, l_2^{(3)} - 0,) &= u_2^{(3)}(t, x_1, l_2^{(1)} - 0), \quad t \in [0, T], \quad x_1 \in [l_1^{(1)}, l_2^{(1)}]; \end{aligned} \quad (2)$$

$$\begin{aligned}
u_0(l_1^{(1)} + 0, x_2, x_3) &= u_1^{(1)}(0 + 0, x_2, x_3), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 2, 3; \\
u_0(x_1, l_1^{(2)} + 0, x_3) &= u_1^{(2)}(0 + 0, x_1, x_3), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 1, 3; \\
u_0(x_1, x_2, l_1^{(3)} + 0) &= u_1^{(3)}(0 + 0, x_1, x_2), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 1, 2; \\
u_0(l_2^{(1)} + 0, x_2, x_3) &= u_2^{(1)}(0 + 0, x_2, x_3), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 2, 3; \\
u_0(x_1, l_2^{(2)} + 0, x_3) &= u_2^{(2)}(0 + 0, x_1, x_3), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 1, 3; \\
u_0(x_1, x_2, l_2^{(3)} + 0) &= u_2^{(3)}(0 + 0, x_1, x_2), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 1, 2; \\
\frac{\partial \rho_0(l_1^{(1)} + 0, x_2, x_3)}{\partial x_1} &= \rho_1^{(1)}(0 + 0, x_2, x_3), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 2, 3; \\
\frac{\partial \rho_0(x_1, l_1^{(2)} + 0, x_3)}{\partial x_2} &= \rho_1^{(2)}(0 + 0, x_1, x_3), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 1, 3; \\
\frac{\partial \rho_0(x_1, x_2, l_1^{(3)} + 0)}{\partial x_3} &= \rho_1^{(3)}(0 + 0, x_1, x_2), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 1, 2; \\
\frac{\partial \rho_0(l_2^{(1)} + 0, x_2, x_3)}{\partial x_1} &= \rho_2^{(1)}(0 + 0, x_2, x_3), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 2, 3; \\
\frac{\partial \rho_0(x_1, l_2^{(2)} + 0, x_3)}{\partial x_2} &= \rho_2^{(2)}(0 + 0, x_1, x_3), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 1, 3; \\
\frac{\partial \rho_0(x_1, x_2, l_2^{(3)} + 0)}{\partial x_3} &= \rho_2^{(3)}(0 + 0, x_1, x_2), \quad x_i \in [l_1^{(i)}, l_2^{(i)}], \quad i = 1, 2;
\end{aligned} \tag{3}$$

where $x = (x_1, x_2, x_3) \in \bar{D} \stackrel{\text{def}}{=} [l_1^{(1)}, l_2^{(1)}] \times [l_1^{(2)}, l_2^{(2)}] \times [l_1^{(3)}, l_2^{(3)}]$, $D = \bar{D} / \partial D$; $l_1^{(i)} (i = \overline{1, 3})$ and $l_2^{(i)} (i = \overline{1, 3})$ are respectively original and finitesimal coordinates for edges of the 3D "parallelepiped" airport area/zone \bar{D} , where expansion of noise pollution is investigated; $t \in [0, T]$, T is the end of the time segment during which the process is investigated; numeric parameters $\varepsilon_i (i = 1, 2)$ are relaxation coefficients; the function $u(t, x) = u(t, x_1, x_2, x_3) \in C^{2,2} \{[0, T] \times \bar{D}\}$ characterizes "quantity" of acoustic pollution (i.e. weighted-noise power) in the spatial point $x \in \bar{D}$ at the time point $t \in [0, T]$, and this function is unknown; the function $\rho(t, x) = \rho(t, x_1, x_2, x_3) \in C^{2,2} \{[0, T] \times \bar{D}\}$ characterizes environment density (i.e. density of the "parallelepiped" area \bar{D}) in the spatial point $x \in \bar{D}$ at the time point $t \in [0, T]$, and this function is also unknown; the initial functions $u_0(x)$, $x \in \bar{D}$ and $\rho_0(x)$, $x \in \bar{D}$ are assumed to be specified, moreover $u_0(x) \in C\{\bar{D}\}$, $\rho_0(x) \in C\{\bar{D}\}$; the boundary functions $u_j^{(i)}(t) \in C\{[0, T]\}$ ($i = \overline{1, 3}; j = 1, 2$), $\rho_j^{(i)}(t) \in C\{[0, T]\}$ ($i = \overline{1, 3}; j = 1, 2$) are also assumed to be a priori prescribed functions; the given function $F(t, x)$ describes an external sources intensity that have an impact on the environment density $\rho(t, x)$ at $(t, x) \in [0, T] \times \bar{D}$.

It is required to unique determine sought functions $u(t, x) \in C^{2,2} \{[0, T] \times \bar{D}\}$ and $\rho(t, x) \in C^{2,2} \{[0, T] \times \bar{D}\}$ from the model (1)-(3)

This section shows the discrete model (special case of model (1)-(3)) of acoustic noise emission, resulting from the continuous model (1)-(3) by its finite-difference approximation of second order.

$$\begin{cases}
 \frac{U_{i,j,k}^{m+2} + 2U_{i,j,k}^{m+1} + U_{i,j,k}^m}{\tau^2} = a_{i,j,k} \cdot \left\{ \frac{U_{i+2,j,k}^m - 2U_{i+1,j,k}^m + U_{i,j,k}^m}{h_1^2} + \frac{U_{i,j+2,k}^m - 2U_{i,j+1,k}^m + U_{i,j,k}^m}{h_2^2} + \frac{U_{i,j,k+2}^m - 2U_{i,j,k+1}^m + U_{i,j,k}^m}{h_3^2} \right\} + \\
 + \rho_{i,j,k}^m + \bar{f}_{i,j,k}^m, \\
 \frac{\rho_{i,j,k}^{m+2} + 2\rho_{i,j,k}^{m+1} + \rho_{i,j,k}^m}{\tau^2} = b_{i,j,k} \cdot \left\{ \frac{\rho_{i+2,j,k}^m - 2\rho_{i+1,j,k}^m + \rho_{i,j,k}^m}{h_1^2} + \frac{\rho_{i,j+2,k}^m - 2\rho_{i,j+1,k}^m + \rho_{i,j,k}^m}{h_2^2} + \frac{\rho_{i,j,k+2}^m - 2\rho_{i,j,k+1}^m + \rho_{i,j,k}^m}{h_3^2} \right\} + \\
 + U_{i,j,k}^m + \bar{f}_{i,j,k}^m;
 \end{cases} \quad (19)$$

Where $U(x_1, x_2, x_3, t) = U_{i,j,k}^m$, $\rho(x_1, x_2, x_3, t) = \rho_{i,j,k}^m$ ($i = \overline{0, N}$; $j = \overline{0, N}$; $k = \overline{0, N}$; $m = \overline{0, T}$);

$h_1 = \frac{\Delta x_1}{N}$; $h_2 = \frac{\Delta x_2}{N}$; $h_3 = \frac{\Delta x_3}{N}$; $\tau = \frac{\Delta t}{N}$; -grid choice for the three spatial coordinates and time respectively.

Discrete model (19) is a clear and monotonic differential scheme and it can be solved numerically, given initial and boundary functions, as well as given certain known $\bar{f}_{i,j,k}^m$ u $\bar{f}_{i,j,k}^m$ ($i = \overline{0, N}$; $j = \overline{0, N}$; $k = \overline{0, N}$; $m = \overline{0, T}$).