MINIMIZING POWER SUM IN TWO-WAY AMPLIFY-AND-FORWARD RELAY CHANNEL

Ebru S. Çetin and Mehmet E. Çelebi

Dept. of Electronic and Communications Engineering
Istanbul Technical University
Maslak-Istanbul, Turkey
e-mail: tokere,mecelebi@itu.edu.tr

ABSTRACT

This paper considers a two-way relay network in which two source nodes exchange messages during two time slots with the help of a half-duplex amplify-and-forward relay node. Closed form expressions of source and relay node powers are obtained by solving an optimization problem which aims to minimize system total power by constraining source nodes' received signal-to-noise ratios (SNR), thus satisfying the minimum quality of service (QoS). Furthermore, the variation of optimum powers versus the parameter which gives information about the channel qualities is investigated for symmetric and asymmetric QoS at the source nodes. In addition, assuming equal channel and noise variances, the effect of variance on the proposed and equal power allocation is illustrated. Numerical results show that the optimum power allocation outperforms the equal power allocation scheme in terms of total power for various variance values and QoS demands.

Index Terms— Amplify-and-forward, two-way relaying, total power minimization

1. INTRODUCTION

Relay networks have recently been the focus of many studies. Although one-way relaying has been extensively studied, it is spectrally inefficient due to the fact that extra channel uses are required for the relay assisted communication of source nodes. On the other hand, two-way relaying proposed by Shannon [1] first, allows two source nodes to simultaneously transmit their signals to the relay by using network coding techniques. Then the relay broadcasts the received signal after some processing such as amplify-and-forward (AF), decode-and-forward, compress-and-forward or estimate-and-forward. Source nodes can recover the desired signal sent by the other one after subtracting its own signal. So, the signal exchange between two source nodes is completed in two time slots, satisfying spectral efficiency.

For practical consideration, AF is desirable due to its non-regenerative processing. Non-regenerative relaying is lower in complexity, processing delay and processing power when compared to regenerative ones. Attracted by the benefits of power allocation, the performance of one-way AF relaying has been extensively studied [2], [3]. For two-way AF relaying, power allocation is considered in [4]-[8]. In [4], the power allocation method maximizing the average sum rate in single relay networks is provided. Minimizing the outage probability is considered in [5]. In [6], optimal power allocation to maximize the minimum data rate is proposed. In [7], sum power minimization satisfying traffic requirements defined by outage probabilities is considered. In [8], total transmit power minimization is discussed in a single relay two-way scheme where beamforming is applied.

In this paper, we consider an AF two-way relaying scenario consisting two source nodes communicating through a relay node, where all nodes equipped with single antenna. We aim to minimize the total power consumption by constraining the received signal to noise ratios (SNR) at the source nodes. Exact closed form expressions for optimum source and relay powers are derived by solving the optimization problem of total power consumption minimization. Power allocation is investigated under two scenarios where symmetric and asymmetric quality of services (QoS) is required at the source nodes. Furthermore, the effect of variance on the power allocation is investigated. Numerical results show that the proposed optimum power allocation scheme outperforms the equal power allocation scheme considerably in terms of total power. It is also noticed that the power saving by the optimum power allocation is more significant for the unbalanced link qualities.

The rest of this paper is organized as follows. Section II describes the system model. Section III formulates the total power optimization problem and provides the derivation of optimum source and relay powers. In Section IV, we comment on the numerical results using the optimum and equal power values. Finally, we state our conclusions in Section V.

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2. SYSTEM MODEL

As shown in Fig. 1, we consider a wireless two-way network where two source nodes, A and B, communicate with each other through a single relay node R. We assume no direct communication between two source nodes because of poor quality of the channel between them. A, B and R are capable of either transmitting or receiving using a half-duplex single-antenna communication scheme. The data symbols x_A and x_B are transmitted by source nodes A and B with powers P_A and P_B, respectively. We assume $E\left\{|x_A|^2\right\} = E\left\{|x_B|^2\right\} = 1$, where $E\{.\}$ stands for the expectation and $|.|$ represents the absolute value of a complex number. The channels between source A and R, and source B and R are denoted by h_A and h_B, respectively. All channels undergo independent and identically distributed (i.i.d.) flat Rayleigh fading and satisfy channel reciprocity.

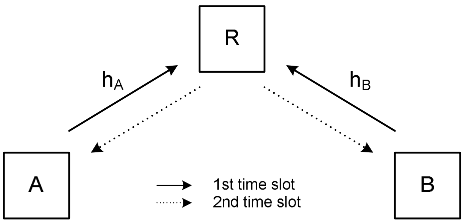

Figure 1. System model for two-way relaying.

We consider total transmission over two time slots. During the first time slot, both sources simultaneously transmit their data to the relay. The signal received at the relay can be represented as

$$y_R = \sqrt{P_A} h_A x_A + \sqrt{P_B} h_B x_B + n_R \tag{1}$$

where n_R is the additive white noise at the relay distributed as $n_R \sim N(0, \sigma_R^2)$. In the second time slot, the relay broadcasts the signal amplified by the factor γ given by

$$\gamma = \sqrt{\frac{P_R}{P_A |h_A|^2 + P_B |h_B|^2 + \sigma_R^2}} \approx \sqrt{\frac{P_R}{P_A |h_A|^2 + P_B |h_B|^2}} \tag{2}$$

where P_R is the transmit power of R. At high signal-to-noise ratio (SNR), the noise becomes negligible, and this amplification factor only relies on the instantaneous channel state information (CSI).

The received signals in the second time slot at source A and B are given, respectively, by

$$\begin{aligned} y_A &= \gamma y_R h_A + n_A \\ &= \gamma \sqrt{P_A} h_A^2 x_A + \gamma \sqrt{P_B} h_A h_B x_B + \gamma h_A n_R + n_A \end{aligned} \tag{3}$$

and

$$\begin{aligned} y_B &= \gamma y_R h_B + n_B \\ &= \gamma \sqrt{P_A} h_A h_B x_A + \gamma \sqrt{P_B} h_B^2 x_B + \gamma h_B n_R + n_B \end{aligned} \tag{4}$$

where n_A and n_B are additive white noise at A and B distributed as $n_A \sim N(0, \sigma_A^2)$ and $n_B \sim N(0, \sigma_B^2)$, respectively. The first term in (3) and the second term in (4) are known as self-interference and can be subtracted from y_A and y_B, respectively. After self-interference cancellation, SNR at source A is

$$SNR_A = \frac{\gamma^2 P_B |h_A|^2 |h_B|^2}{\gamma^2 |h_A|^2 \sigma_R^2 + \sigma_A^2}. \tag{5}$$

Similarly, the SNR at source B is

$$SNR_B = \frac{\gamma^2 P_A |h_A|^2 |h_B|^2}{\gamma^2 |h_B|^2 \sigma_R^2 + \sigma_B^2}. \tag{6}$$

Substituting (2) into (5) and (6) and assuming $\sigma_A^2 = \sigma_B^2 = \sigma_R^2 = \sigma^2$, the SNRs at A and B are given respectively by

$$SNR_A = \frac{|h_A|^2 |h_B|^2 P_R P_B}{|h_A|^2 \sigma^2 P_R + |h_A|^2 \sigma^2 P_A + |h_B|^2 \sigma^2 P_B} \tag{7}$$

and

$$SNR_B = \frac{|h_A|^2 |h_B|^2 P_R P_A}{|h_B|^2 \sigma^2 P_R + |h_A|^2 \sigma^2 P_A + |h_B|^2 \sigma^2 P_B}. \tag{8}$$

3. POWER OPTIMIZATION

In this section, we are interested in optimizing powers allocated to both source nodes and the relay to satisfy the minimization of total power consumption while maintaining a lower bound for the SNRs at sources A and B in the two-way system. The optimization problem can be formulated as

$$
\begin{aligned}
&\min_{P_A, P_B, P_R} \; P_A + P_B + P_R \\
&\textit{subject to} \quad SNR_A \ge \psi_A \\
&\qquad\qquad\quad\;\; SNR_B \ge \psi_B \\
&\qquad\qquad\quad\;\; 0 \le P_A,\ 0 \le P_B,\ 0 \le P_R
\end{aligned} \tag{9}
$$

where SNR_A is lower bounded by ψ_A and similarly, SNR_B by ψ_B. Rewriting the lower bound expressions on SNR_A and SNR_B, the first and second inequalities in (9) become

$$
\begin{aligned}
&\sigma^2 \psi_A (|h_A|^2 P_A + |h_B|^2 P_B + |h_A|^2 P_R) \\
&\qquad - |h_A|^2 |h_B|^2 P_B P_R \le 0
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
&\sigma^2 \psi_B (|h_A|^2 P_A + |h_B|^2 P_B + |h_B|^2 P_R) \\
&\qquad - |h_A|^2 |h_B|^2 P_A P_R \le 0
\end{aligned} \tag{11}
$$

respectively. Using (10) and (11), the optimization problem in (9) is reformulated by

$$
\begin{aligned}
&\min_{P_A, P_B, P_R} \; P_A + P_B + P_R \\
&\textit{subject to} \quad \sigma^2 \psi_A (|h_A|^2 P_A + |h_B|^2 P_B + |h_A|^2 P_R) \\
&\qquad\qquad\qquad - |h_A|^2 |h_B|^2 P_B P_R \le 0 \\
&\qquad\qquad\quad\;\; \sigma^2 \psi_B (|h_A|^2 P_A + |h_B|^2 P_B + |h_B|^2 P_R) \\
&\qquad\qquad\qquad - |h_A|^2 |h_B|^2 P_A P_R \le 0 \\
&\qquad\qquad\quad\;\; 0 \le P_A,\ 0 \le P_B,\ 0 \le P_R.
\end{aligned} \tag{12}
$$

The Lagrangian function for (12) is then given by

$$
\begin{aligned}
&L(P_A, P_B, P_R, \mu_1, \mu_2, \mu_{11}, \mu_{22}, \mu_{33}) \\
&= (P_A + P_B + P_R) \\
&+ \mu_1 (\sigma^2 \psi_A (|h_A|^2 P_A + |h_B|^2 P_B + |h_A|^2 P_R) - |h_A|^2 |h_B|^2 P_B P_R) \\
&+ \mu_2 (\sigma^2 \psi_B (|h_A|^2 P_A + |h_B|^2 P_B + |h_B|^2 P_R) - |h_A|^2 |h_B|^2 P_A P_R) \\
&- \mu_{11} P_A - \mu_{22} P_B - \mu_{33} P_R
\end{aligned} \tag{13}
$$

where μ_1, μ_2, μ_{11}, μ_{22} and μ_{33} are Karush-Kuhn-Tucker (KKT) multipliers. Applying Karush-Kuhn-Tucker (KKT) conditions for Lagrangian optimality, we can obtain the following equalities.

$$
\begin{aligned}
&\partial L / \partial P_A = 0 \\
&\partial L / \partial P_B = 0 \\
&\partial L / \partial P_R = 0 \\
&\mu_1 (\sigma^2 \psi_A (|h_A|^2 P_A + |h_B|^2 P_B + |h_A|^2 P_R) - |h_A|^2 |h_B|^2 P_B P_R) = 0 \\
&\mu_2 (\sigma^2 \psi_B (|h_A|^2 P_A + |h_B|^2 P_B + |h_B|^2 P_R) - |h_A|^2 |h_B|^2 P_A P_R) = 0 \\
&\mu_{11} P_A = 0, \quad \mu_{22} P_B = 0, \quad \mu_{33} P_R = 0 \\
&\mu_1 \ge 0 \;\; \mu_2 \ge 0 \;\; \mu_{11} \ge 0 \;\; \mu_{22} \ge 0 \;\; \mu_{33} \ge 0
\end{aligned} \tag{14}
$$

Solving the nonlinear equation sets given in (14) and defining

$$
Q = 1 \Big/ \Big(|h_B| \big(|h_A| \psi_B + |h_B| \psi_A \big) \Big) \tag{15}
$$

for notational simplicity, we can obtain the optimal solutions given by (16) through (21).

$$
\begin{aligned}
P_A &= -\frac{\sigma^2 \psi_B \left(|h_A|^2 - |h_B|^2 \right) \left(Q |h_B|^2 \psi_A - 1 \right)}{|h_A|^2 |h_B|^2 \left(Q |h_A|^2 \psi_B + Q |h_B|^2 \psi_A - 1 \right)} \\
&= \frac{\sigma^2 \psi_B \left(|h_A| + |h_B| \right)}{|h_A|^2 |h_B|}
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
P_B &= \frac{\sigma^2 \psi_A \psi_B Q \left(|h_A|^2 - |h_B|^2 \right)}{|h_B|^2 \left(Q |h_A|^2 \psi_B + Q |h_B|^2 \psi_A - 1 \right)} \\
&= \frac{\sigma^2 \psi_A \left(|h_A| + |h_B| \right)}{|h_A| |h_B|^2}
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
P_R &= -\frac{\sigma^2 \psi_B \left(|h_A|^2 - |h_B|^2 \right)}{|h_A|^2 |h_B|^2 \left(Q |h_B|^2 \psi_B + Q |h_B|^2 \psi_A - 1 \right)} \\
&= \frac{\sigma^2 \left(|h_A| + |h_B| \right) \left(|h_A| \psi_B + |h_B| \psi_A \right)}{|h_A|^2 |h_B|^2}
\end{aligned} \tag{18}
$$

$$
P_{total} = \frac{\sigma^2 \left(|h_A|^2 + |h_B|^2 + 2 |h_A| |h_B| \right) (\psi_A + \psi_B)}{|h_A|^2 |h_B|^2} \tag{19}
$$

$$
\begin{aligned}
\mu_1 &= \frac{\left\{ \begin{array}{l} Q |h_A|^2 |h_B|^2 \psi_B{}^2 + Q |h_B|^4 \psi_A{}^2 \\ +2Q |h_A|^2 |h_B|^2 \psi_A \psi_B - |h_A|^2 \psi_B - |h_B|^2 \psi_A \end{array} \right\}}{\left\{ \begin{array}{l} \sigma^2 |h_A|^2 \left(Q |h_B|^2 \psi_A + Q |h_B|^2 \psi_B - 1 \right) \\ \times \left(|h_A|^2 \psi_B{}^2 - |h_B|^2 \psi_A{}^2 \right) \end{array} \right\}} \\
&= \frac{\left(|h_A| |h_B|^2 \psi_A{}^2 + |h_A|^3 \psi_B{}^2 - 2 \psi_A \psi_B |h_A|^2 |h_B| \right)}{\sigma^2 |h_A|^2 \left(|h_A|^2 \psi_B{}^2 - |h_B|^2 \psi_A{}^2 \right) \left(|h_A| \psi_B - |h_B| \psi_A \right)}
\end{aligned} \tag{20}
$$

$$
\mu_2 = \frac{Q}{\sigma^2} = \frac{1}{\sigma^2 |h_B| \left(|h_A| \psi_B + |h_B| \psi_A \right)} \tag{21}
$$

4. NUMERICAL RESULTS

In this section, we present the numerical results of the proposed power allocation scheme under parameters ψ_A, ψ_B

and k. For simplicity, we assume that $|h_A||h_B|=1$ and $|h_B|/|h_A|=k$.

In Fig. 2, the lower bound of SNR_A is chosen to be $\psi_A=1$ and the variances $\sigma^2=0.5$. In this case, we plot optimum power variation of source and relay nodes versus $\log(k)$ for two different scenarios: i) $\psi_B=\psi_A$ and ii) $\psi_B=5\times\psi_A$. It can be observed that P_A increases as k changes from 0.1 to 10 while P_B decreases, since the quality of the channel between source B and R improves as opposed to the channel between source A and R. In the scenario (i), for k values smaller than 1, more transmit power is consumed at the node B, since the channel between node B and R is poor. On the contrary, as k increases from 1 to 10, P_B is less than P_A. When k equals to 1, powers allocated to nodes A and B are equal because of the same quality of channels from nodes A and B to the relay. It is observed that the powers P_A and P_B are symmetric with respect to the value $k=1$. In addition, P_R decreases as k changes from 0.1 to 1, while it increases as k changes from 1 to 10. As a result, the total power P_{total} is convex against the ratio k. In the scenario (ii), more quality of service is aimed at source B via increasing the lower bound on the SNR_B. Therefore, the symmetry of powers P_A and P_B is achieved at smaller values of k and similarly the minimum relay power P_R is obtained for larger values of k. In conclusion, the asymmetry of service qualities at different source nodes causes the change in the symmetry of source and relay powers versus k in different directions.

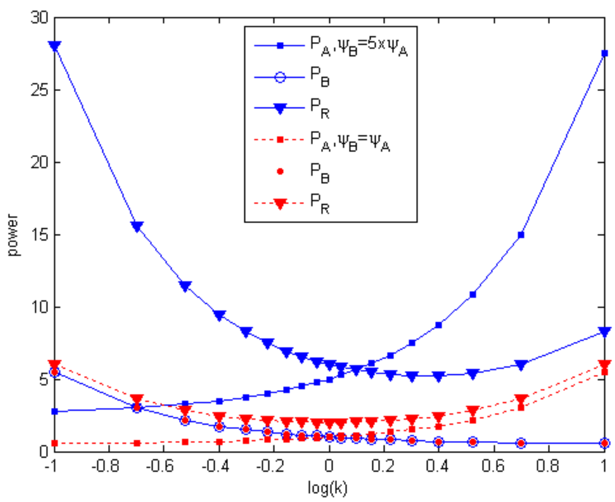

Figure 2. Optimum power allocation when $\psi_A=\psi_B$ and $\psi_B=5\times\psi_A$ for $\psi_A=1$ and $\sigma^2=0.5$.

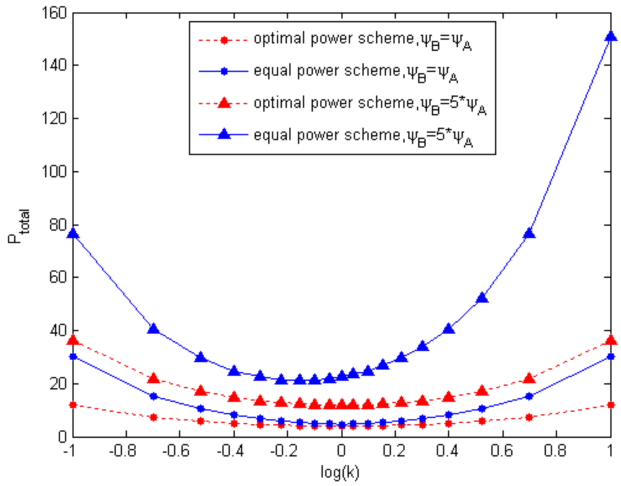

Figure 3. Total power in equal and optimum power allocation schemes versus $\log(k)$ when $\psi_A=\psi_B$ and $\psi_B=5\times\psi_A$ for $\psi_A=1$ and $\sigma^2=0.5$.

Fig. 3 compares total power of optimum and equal power allocation schemes under scenarios (i) and (ii). Our proposed power allocation scheme provides a significant power saving against equal power allocation for both scenarios. The advantage over equal power allocation scheme is clearer when the gap between the qualities of channels widens: this is due to the fact that in the case of unbalanced links, the weaker link will dominate the performance of the two-way relay channel and will consume too much power to maintain the traffic requirements. We also observe in Fig. 3 that, there is a constant gap between optimum power allocation schemes in total power for two scenarios. When we need to improve the quality of service at either one of the sources, more total power is consumed as expected. However, the gap between equal and optimum power allocation schemes in terms of total power widens in scenario (ii) in order to satisfy the asymmetric traffic requirements at different source nodes. For the given ψ_A, ψ_B values in scenario (ii), we aim to have more QoS at source B, thus P_A will dominate the total power as the channel between source A and R gets poor. Clearly, this leads to the asymmetry of total power versus $\log(k)$ in the equal power allocation scheme. On the other hand, the symmery of total power in both equal and total power allocation schemes is obvious for scenario (i).

Fig. 4 presents the optimum power allocation under two different σ^2 values. As can be seen from this figure, the change in the variance is insignificant for the symmetry of powers versus $\log(k)$. However, when the variance becomes larger, the total power consumption increases as expected. In addition, as k increases, we can observe that increasing σ^2 value make the increment in P_A and the decrement in P_B slightly sharper. On the other hand, the convexity of the P_R change becomes clearer.

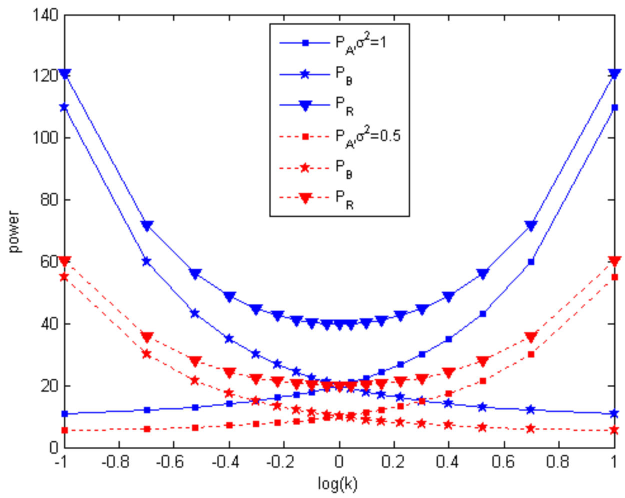

Figure 4. Optimum power allocation when $\psi_A = \psi_B = 10$ and σ^2 is fixed to 0.5 and 1.

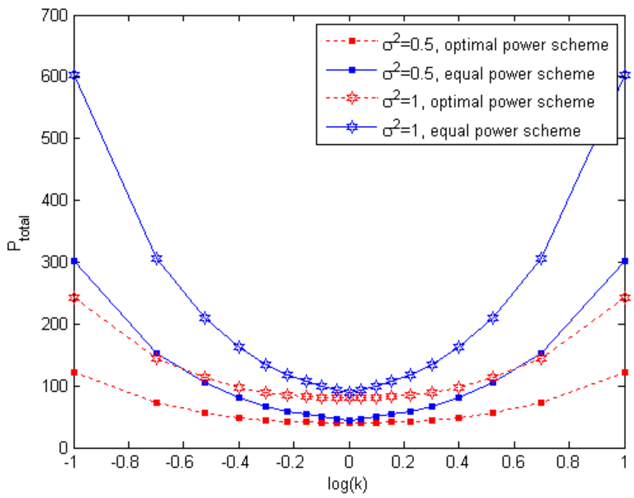

Figure 5. Total power in equal and optimum power allocation schemes versus $\log(k)$ when $\psi_A = \psi_B = 10$ and σ^2 is fixed to 0.5 and 1.

We plot Fig. 5 to show how effective our proposed power allocation scheme against the equal power allocation scheme under various σ^2 values. Since the symmetry in the source and relay powers is maintained, the symmetry of total power is not affected from the change of variance. Furthermore, in the case of unbalanced links, power saving due to the optimum power allocation is significant. For balanced links where the channel qualities becomes more closer, the advantage over equal power allocation has slight importance.

5. CONCLUSIONS

In this paper, we studied the power allocation in two-way AF single relay channels. Closed form expressions for the optimum powers are derived by solving the optimization problem including total power minimization where the received SNRs at source nodes are lower-bounded, thus achieving minimum quality of service. We evaluated the variation of optimum powers against the parameter k which quantifies the relative channel qualities. Under symmetric and asymmetric quality of service at the source nodes, it is shown that optimum power allocation outperforms equal power allocation scheme in terms of total power. For the unbalanced links, the advantage over equal power allocation is more noticeable. Furthermore, assuming equal channel and noise variances, the symmetry of powers is not affected from the variance deviations. Numerical results show that optimum power allocation always outperforms equal power allocation scheme.

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