BROADBAND LOGARITHMICALLY PERIODIC ANTENNA STRUCTURES*

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Summary

Antenna structures for which the input impedance and radiation patterns vary periodically with the logarithm of the frequency are described. For a particular class of these structures the variation of the electrical characteristics over a period is negligible, the result being an antenna for which the impedance and patterns are essentially independent of frequency over bandwidths greater than ten to one. The antennas are linearly polarized and bi-directional beams of approximately equal principal plane beamwidths are obtained. The beamwidth may be controlled to a considerable extent by the geometry of the structure.

Introduction

The design of broadband antennas has been the subject of many investigations over the years. Although some success has been achieved in obtaining broadband impedance characteristics, it is only recently that broadband impedance and pattern characteristics have been obtained. The use of the term broadband has been rather loose. It has, in the past, been used to describe antennas operating over a 2 or 3 to 1 band even though their patterns and impedance may vary widely over that range. Its use here is intended to have the meaning of performance which is essentially independent of frequency over the band of frequencies considered.

This investigation has been concerned with a new approach to the design of broadband antennas. The method consists of designing the antenna structure so that its characteristics are periodic with the logarithm of the frequency. If, then, the variation of the antenna characteristics over a single period are small or negligible the result is a broadband antenna. Although this may appear to be a backward approach, its merits will become evident when the results are examined. Suffice it to say, at this point, that bandwidths of ten to one have been obtained by this method with evidence that even much greater bandwidths are possible.

In the following sections, some design principles for broadband antennas will be described and the experimental results for logarithmically periodic structures will be presented.

Design Principles

The antennas described in this report embody three basic design principles. The first of these is the "angle" concept which is a design approach wherein the geometry of the antenna structure is described, so far as is practical, by angles rather than lengths. The second principle makes use of the fact that the input impedance of an antenna identical to its complement is independent of frequency. These first two principles are the subject of the preceding paper[1] and will be mentioned only briefly here. The third, which will be introduced in this report, consists of designing the antenna structure such that its electrical properties repeat periodically with the logarithm of the frequency.

An antenna described completely by angles, such as an infinite biconical antenna, would make an ideal broadband radiator since its operation is independent of frequency. In practice, however, the antenna must be of finite length and although the input impedance approaches a constant value with increasing frequency the radiation characteristics usually vary considerably due to what has been termed the "end effect". To date the most successful broadband angular antenna is the logarithmic spiral.[2, 3] Experimental results, over a ten to one bandwidth, have demonstrated that the patterns and impedance are essentially constant except for a rotation of the pattern with frequency. Apparently the "end effect" (or the effect of finite length) is negligible for the logarithmic spiral antenna.

The relationship of the impedance of a plane metal sheet antenna to its complementary slot antenna yields the basis of the second design-principle. If the shape is such that the metal sheet antenna is equal to its complement it will have a theoretical input impedance of 60π ohms, this result being independent of frequency. This gives a design approach for an antenna having a constant input impedance and although it also requires a structure of infinite extent it puts no restrictions on the shape of the elements other than the identical complement condition. A few examples of shapes having identical complements are shown in a figure of reference 1. These are generated by dividing a plane into four equal quadrants, the areas of which are alternately conductor and free space. The lines dividing the quadrants may take any shape whatsoever and a condition for equal

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complement is satisfied if when any dividing line is rotated 90°, about the center, it is coincident with the adjacent dividing line.

A logarithmically periodic antenna is defined here as an antenna structure for which the electrical properties vary periodically with the logarithm of the frequency. As will be shown, the logarithmically periodic antenna is a simple modification of an angular antenna. This modification is successfully used to reduce the "end effect". Although the modification causes, in general, a variation of the electrical properties with frequency, the magnitude of the variation is sometimes very small, the result being a broadband antenna.

A precise definition of logarithmically periodic structures may be obtained by considering the transformation

$$z = \ln w$$

where w and z are complex numbers. Letting $w = \rho e^{j\Theta}$ and $z = x + jy$, it is easily shown that

$$\rho = e^{x} \text{ or } x = \ln \rho$$
$$\Theta = y$$

With this transformation, circles and radial lines in the w plane are mapped into vertical and horizontal lines, respectively, in the z plane.

Let us consider now how the angular structures, in the w plane, are mapped into the z plane. As shown in figure 1, the "bow tie" antenna is transformed into horizontal parallel strips and the equiangular spiral antenna is transformed into inclined parallel strips. A logarithmically periodic structure is formed by introducing periodic variations on the parallel strips in the z plane and then transforming to the w plane. A few examples are shown in figure 2. Figures 2(a) and 2(b) show logarithmically periodic slot and tooth structures. The teeth in figure 2(c) are formed by sinusoidal curves in the z plane. It is also possible to introduce logarithmically periodic variations in the equiangular spiral but the usefulness of this is doubtful because the "end effect" for the spiral is practically negligible. It will be noticed that in the w plane all dimensions involved in the definition of a logarithmically periodic structure are proportional to their distance from the origin or feed point. In figure 2(a), the slots are bounded by the radii R_n, r_n and the subtended angle β. The radii R_n-1, R_n, R_n+1, . . . form a geometric sequence of terms where the geometric ratio is defined by

$$\tau = \frac{R_{n+1}}{R_n} .$$

The radii r_{n-1}, r_n, r_{n+1},. . . form a similar sequence having the same geometric ratio. The width of the slot is defined by

$$\sigma = \frac{r_n}{R_n} .$$

It can be seen that infinite structures of this type have the property that, when energized at the vertex, the fields at a frequency f will be repeated at all other frequencies given by $\tau^n f$ (apart from a change of scale) where n may take on any integral value. When plotted on a logarithmic scale, these frequencies are equally spaced with a separation or period of $\ln \tau$; hence the name logarithmically periodic structures.

By probing the fields along several logarithmically periodic antenna structures, it has been found that the fields decay very rapidly after passing the point where a resonant discontinuity exists. The slots in figure 2(a) which are approximately a half-wavelength long and the teeth in figure 2(b) which are approximately one quarter wavelength long are considered to be resonant discontinuities. This decay of the fields past the point of a resonant discontinuity has been found to cause the "end effect" to be small or negligible for all logarithmically periodic structures tested.

There are an unlimited number of different logarithmically periodic antenna configurations possible. Because of the limited time available only structures having tooth-like discontinuities similar to that of figure 2(b) have been investigated to date.

Experimental Results

Description of Experimental Models

The antenna models investigated were constructed as shown in figure 3. The conducting sheet, from which the antennas were cut, was of light gauge copper sheet (approx. .020 inches thick). For extremely broad band applications, the thickness of the sheet should be made proportional to the distance from the vertex. The periodic discontinuities are in the form of teeth connected to a triangular conducting strip. The ratios τ and σ are determined as in the preceding section and it is seen that in order for the structure to be equal to its complement (in the infinite case), the requirement that the sum of the angles α and β be equal to 90° is necessary. A small area near the center was left as solid conductor since continuation of the obstacles, in that direction, requires an infinite number of teeth of zero width in the limit. The smallest teeth near the center will determine the upper frequency limit of periodic operation while the radius R_1 and presumably the angles α and β (length of teeth) will determine the lower frequency limit. For all models tested the ratio σ was taken equal to the square root of the ratio τ providing a ratio of tooth to slot width which is the same for all rows of teeth. The structure was fed across the vertex with a coaxial line laying on one of the triangular sheets. The outer conductor was bonded to the sheet and the inner conductor was extended across the vertex and connected to the other triangular sheet.

Performance of a Logarithmically Periodic Structure Identical to its Complement

A model of the type shown in figure 3 was constructed with the following values affixed to the variables:

$$R_1 = 10 \text{ inches}$$
$$\tau = .81$$
$$\sigma = \sqrt{\tau} = .9$$
$$\alpha = 45°$$
$$\beta = 45°.$$

The input impedance of the antenna was measured over the band 400 to 1600 mc. Figure 4 shows the results, plotted on a Smith impedance diagram, referred to the feed point and relative to 50 ohms. The input impedance is seen to be fairly constant with the locus of points centered about a resistance level of approximately 150 ohms. The discrepancy between this value and that of the predicted 60π ohms (189 ohms) is assumed to be due to the finite thickness and length of the elements, the presence of the feed cable and the unavoidable introduction of some series reactance at the feed point.

Radiation patterns of this structure were measured and found to be of fairly uniform shape over a better than 10 to 1 band. Figure 5 defines the coordinate system and figure 6 shows patterns in the two principal planes measured at frequencies of 1525 and 1700 mc. These patterns were found to be typical of the patterns in both planes at other frequencies. For the purpose of demonstrating the bandwidth obtained with this laboratory model, radiation patterns for the plane $\phi = 0$ are shown in figures 7 through 9 which cover a better than ten to one bandwidth. The pattern at 5000 mc (figure 9d) is seen to have degenerated somewhat in symmetry and this is felt to be due to the difficulty in maintaining perfectly coplanar elements when using such thin material for the construction.

Contrary to what might be expected, the maximum radiated power (along the z axis) has its electric field polarized in the yz plane, whereas for a similar structure, without the periodic obstacles, the normal polarization would be in the xz plane.

Performance with Variation of Tooth Angle α

Several additional models were measured which had tooth angles ranging from $\alpha = 20$ degrees to $\alpha = 60$ degrees. In each case the angle β was adjusted so that the sum of α and β was 90 degrees and the structure was thus maintained equal to its complement. The ratio τ was .81 for all models and σ was held equal to $\sqrt{\tau}$.

Input impedance and radiation patterns were measured for each of the antennas constructed. The impedance of each model was found to be essentially the same as that shown in figure 4, the only significant differences being the low frequency limits of constant input impedance. It was found that as the tooth angle α was decreased, holding R_1 fixed, the tooth lengths become shorter and the low frequency limit was raised. For example, the low frequency limit for nearly constant input impedance was approximately 800 mc for the structure with 20 degrees of teeth, whereas it was approximately 400 mc for 45 degrees of teeth. Radiation patterns, in the plane $\phi = 0$, typical of each antenna tested are shown in figures 10 through 12. Examination of these patterns indicates that the performance of the antenna is not peculiar to a particular tooth angle; in fact, the patterns are remarkably similar for all models tested. The largest observed differences are found in the relative amplitude of the E_Θ polarization which is seen, in general, to increase with decreasing tooth angle. This is to be expected since if the angle α went to 0 degrees the radiation would be entirely of E_Θ polarization. It is also observed that the E_ϕ patterns for $\alpha = 20$ degrees have degenerated somewhat in shape and this is also as would be expected.

Performance with Variation of the Ratio τ

The parameter τ determines, what may be considered, the bandwidth of a period of operation. That is,

$$\frac{f_1}{f_2} = \tau$$

where f_1 and f_2 are two frequencies exactly one period apart ($f_2 > f_1$). If the bandwidth of a period can be increased without causing significant variations in the radiation patterns, then the ratio τ can be decreased and the antenna will retain its broadband properties. One advantage to decreasing τ is that the number of teeth required between any two radii of the structure is reduced and construction is simpler near the feed point. That is, the teeth become fewer and have greater separation for the same values of R_1 and the minimum tooth spacings could become an important consideration for high power operation.

Models of the logarithmically periodic structure, of the type shown in figure 3, having geometric ratios τ equal to .70, .50, .25, and .15 were constructed. In each case the ratio σ was taken equal to $\sqrt{\tau}$ and the structure was made identical to its complement. The angles α and β were each 45 degrees and the radius R_1 was maintained at ten inches.

Radiation patterns of the models having τ equal to .70, .50, and .25 were measured and are shown in figures 13 through 16. Inspection of these patterns show the most significant effect of reducing the ratio τ to be an increase of the directivity of the antenna in both principal planes. The average half power beamwidths (this is the average of both principal plane patterns for several frequencies over the band) for the various values of τ are listed in table I.

TABLE I.
AVERAGE H. P. BEAMWIDTH VS. THE PARAMETER τ

τ	Average H. P. Beamwidth in Degrees
.81	73
.70	70
.50	55
.25	38

This result was an unexpected one and increases to a considerable extent the flexibility of logarithmically periodic antenna structure of this type. The ratio τ becomes a parameter for controlling the beamwidth of the antenna without affecting its other characteristics. Since an increase in directivity generally corresponds to an increase in physical aperture size, it is to be expected that the lower limit of frequency independent operation, for a given radius R_1, will be raised. This was found to be the case although not demonstrated by the data presented. For example, the antenna having τ equal to .25 has fairly constant half-power beamwidths of approximately forty degrees, as shown, from 900 mc/sec up in frequency. The beamwidth, however, was found to broaden rapidly below 900 mc/sec to approximately seventy degrees at 700 mc/sec. The frequency of 900 mc/sec is thus considered the low limit of frequency independent operation for the antenna having τ equal to .25 and R_1 equal to ten inches. The low frequency limits of antennas having τ equal to .70 and .50 with R_1 equal to ten inches were not investigated but are expected to be between 450 and 900 mc/sec, which represent these limits for τ equal to .81 and .25 respectively. The result of this is simply that, in order to increase the directivity of the antenna for a given band of frequencies, the minimum size of the aperture must be increased as well as reducing the ratio τ.

The patterns shown in figure 17 are for τ equal to .15. These patterns were measured in the plane $\phi = 0°$ and as is seen, the performance has deteriorated somewhat. The E_θ polarization is much higher at some frequencies and the general shape of the pattern is seen to vary.

The effect of varying the tooth angle has not been investigated for τ of ratios other than .81 but it is expected that similar changes in performance characteristics would occur.

Periodic Structures Not Having Identical Complement

All of the antennas discussed thus far have had shapes identical to their complements and, as a result, have had input impedances near 180 ohms. For the best match then, the line feeding the antenna must have a characteristic impedance near this value. It would be desirable if the input impedance of the periodic structure could be varied to match its characteristic impedance to that of available transmission lines.

A model of the periodic antenna was constructed in order to determine whether the input resistance level could be reduced without interfering with the broadband properties. This was accomplished by increasing the angle β without changing α. The expected result of this would be an increase in the capacity of the structure and a corresponding decrease in the characteristic impedance. The model tested was constructed with the following values for the various parameters,

$$R_1 = 10 \text{ inches}$$
$$\tau = .81$$
$$\sigma = .9$$
$$\alpha = 45°$$
$$\beta = 75°$$

Since the sum of the angles α and β is not equal to 90°, the antenna is not equal to its complement and the input impedance would be expected to be some value other than 189 ohms.

It was found that the mean resistance level had been reduced to approximately 100 ohms, but that the impedance locus had spread out. This result led to the conclusion that it would be best to use a structure identical to its complement and fed by a wide band tapered or stepped transformer.[4, 5] Radiation patterns were measured over the frequency band 1000 to 1500 mc covering more than one period of the structure. No degradation of the patterns, compared to the identical complement antennas was observed.

Conclusions

The experimental results obtained during this investigation demonstrate a promising new approach to the design of antennas whose performance is essentially independent of frequency. The method described provides a means of obtaining linearly polarized, bidirectional antennas having approximately equal and constant principal plane beamwidths over bandwidths of greater than ten to one. The directivity, which is essentially constant for a given geometry, can be varied over a substantial range by the alteration of a single parameter. This result is of particular significance since it provides a means of controlling the beamwidth of the antenna separate from other considerations with the exception of overall size for a given band.

The major portion of this work has been restricted to investigation of two dimensional structures of the type shown in figure 3. This class of antenna structures displays remarkably broadband characteristics over large ranges in the various design parameters but represents only one of an unlimited number of possible configurations which embody the logarithmically periodic concept. There is, at this time, no evidence to suggest that the structures of the particular type investigated are of optimum shape to give the best broadband results. It is possible that improved performance or perhaps some new characteristics are available through variations in the general configuration. At least one such possible variation is suggested in figure 2(c). The design principles can also be extended to three dimensional structures.

Acknowledgement

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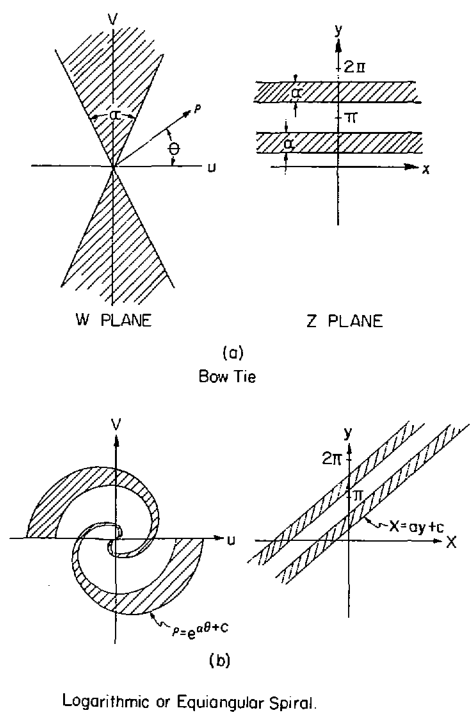

Fig. 1
Angular structures.

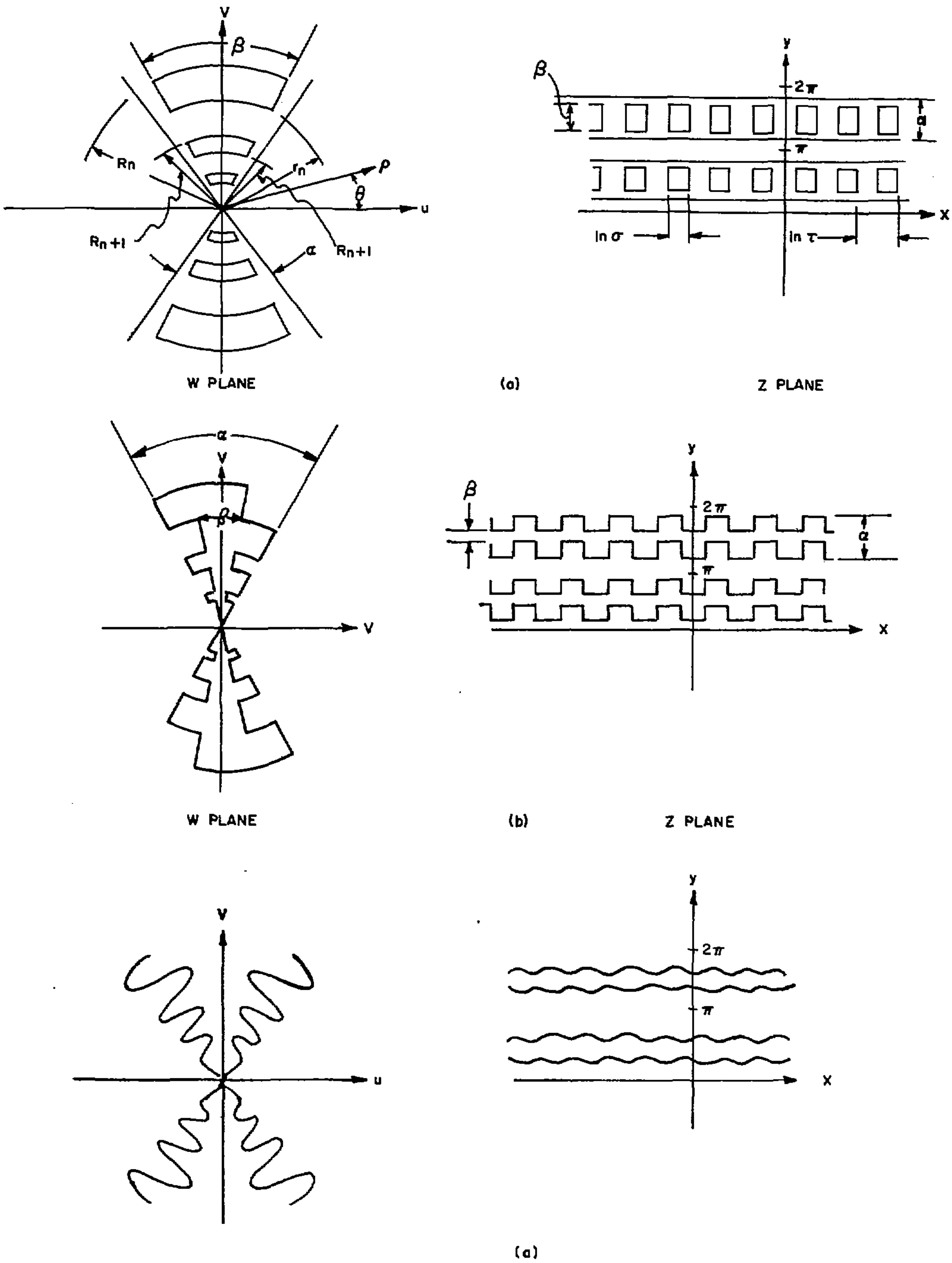

Fig. 2
Logarithmically periodic structures.

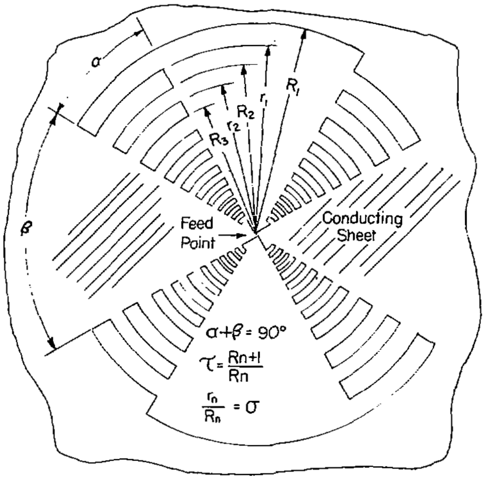

Fig. 3
Experimental model configuration.

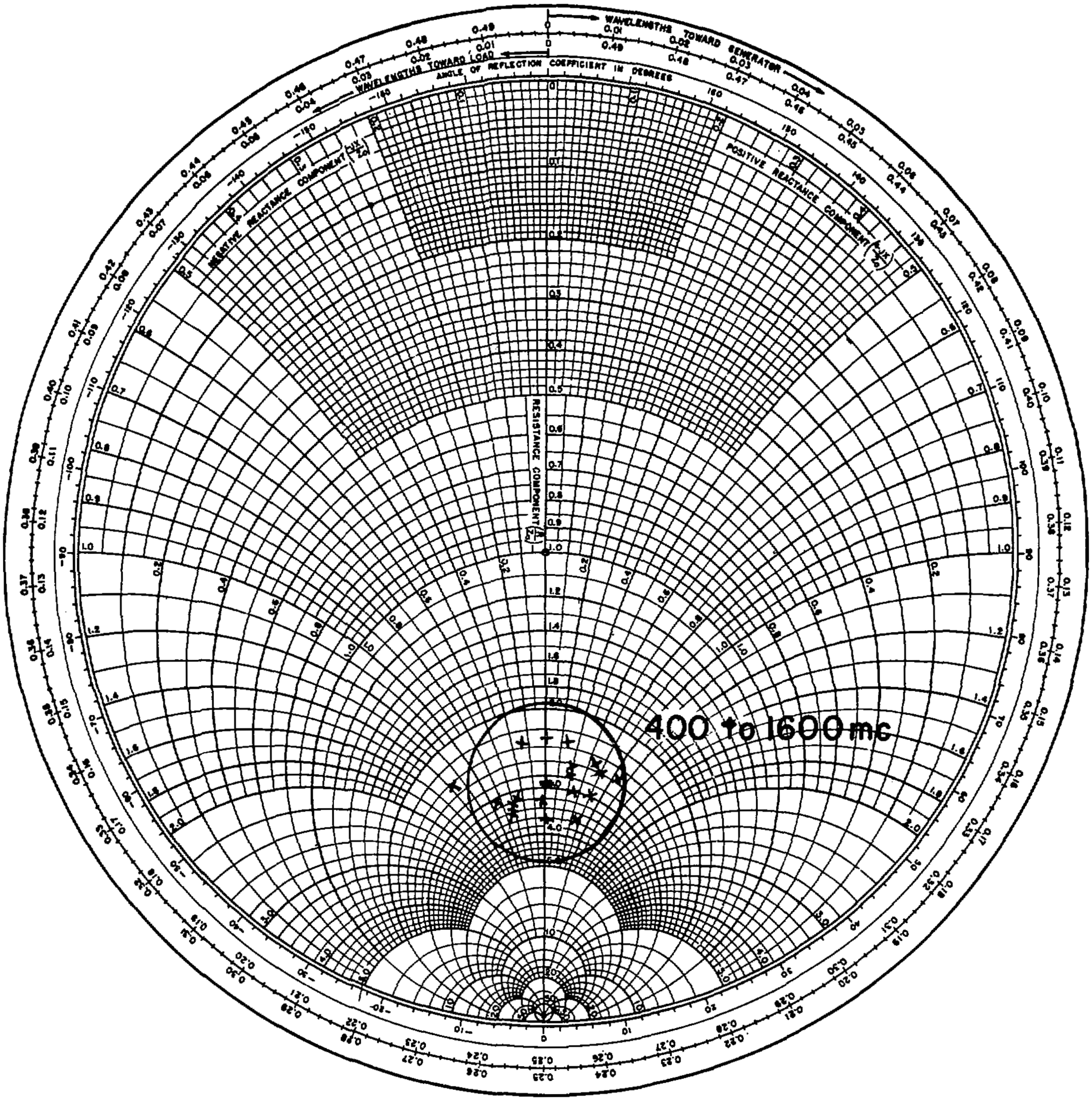

Fig. 4
Input impedance of self complementary model.

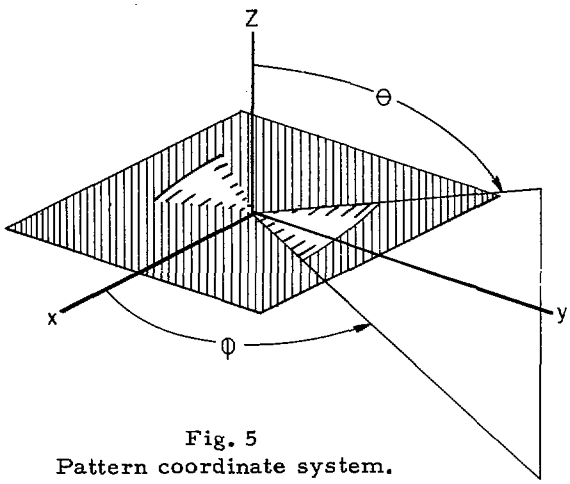

Fig. 5
Pattern coordinate system.

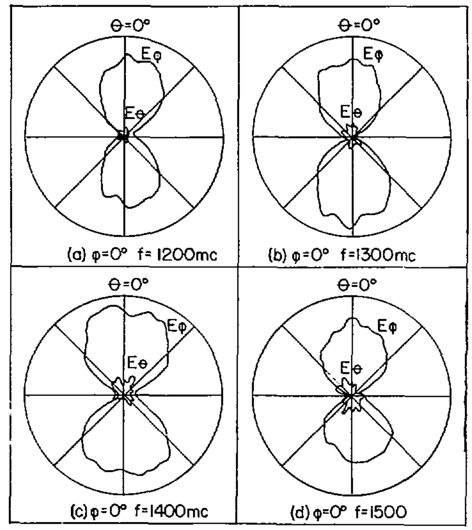

Fig. 8
Radiation patterns for $\alpha=\beta= 45^{\circ}$, $\tau= .81$.

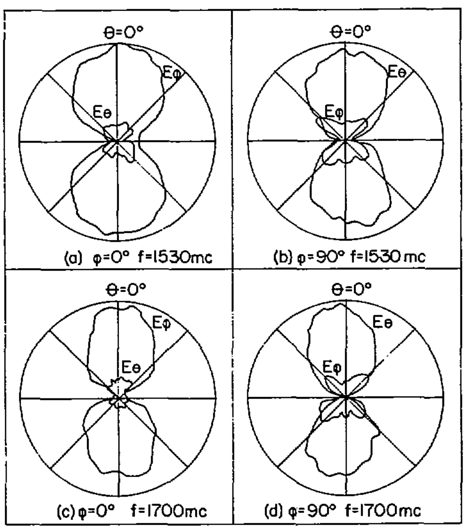

Fig. 6
Typical principal plane radiation patterns for $\alpha=\beta=45^{\circ}$, $\tau= .81$.

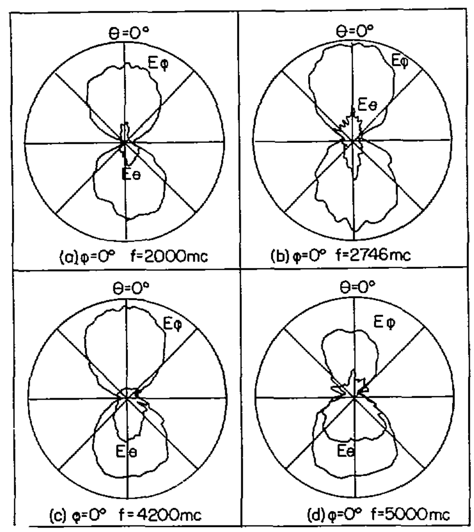

Fig. 9
Radiation patterns for $\alpha=\beta= 45^{\circ}$, $\tau= .81$.

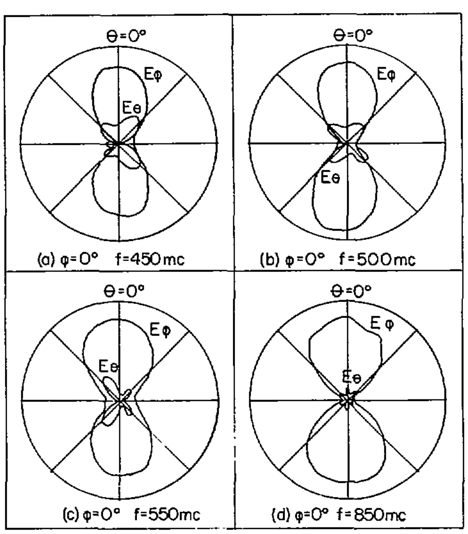

Fig. 7
Radiation patterns for $\alpha=\beta= 45^{\circ}$, $\tau= .81$.

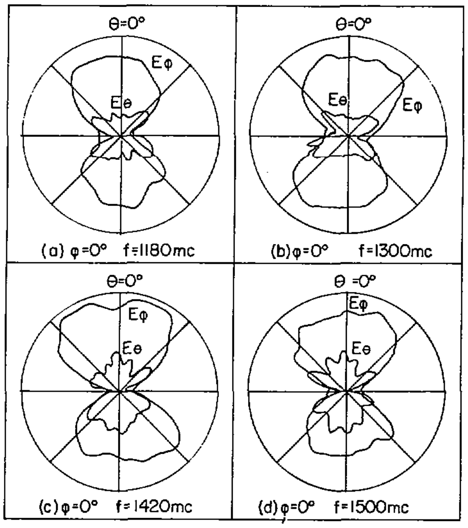

Fig. 10
Radiation patterns for $\alpha = 20^o$ $\beta = 70^o$ $\tau = .81$.

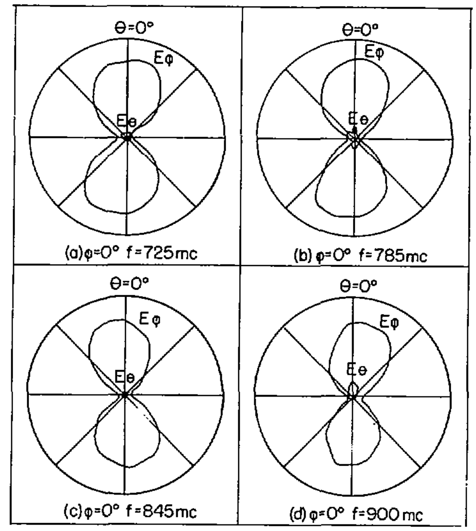

Fig. 12
Radiation patterns for $\alpha = 60^o$ $\beta = 30^o$ $\tau = .81$.

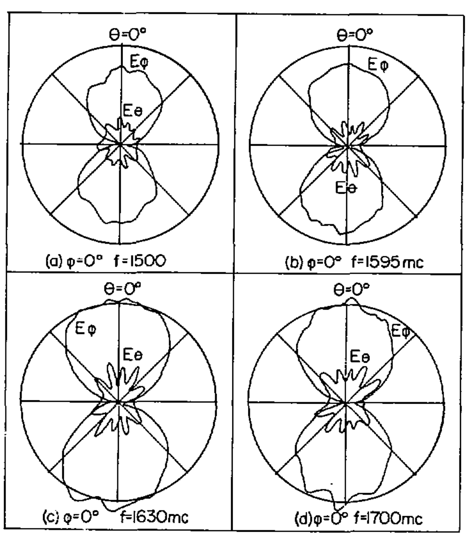

Fig. 11
Radiation patterns for $\alpha = 30^o$ $\beta = 60^o$ $\tau = .81$.

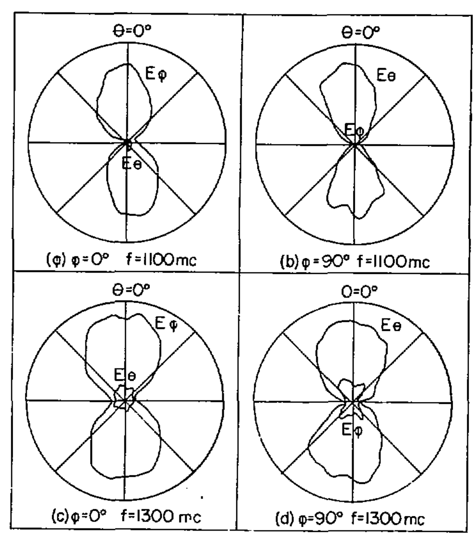

Fig. 13
Radiation patterns $\alpha = \beta = 45^o$ $\tau = .70$

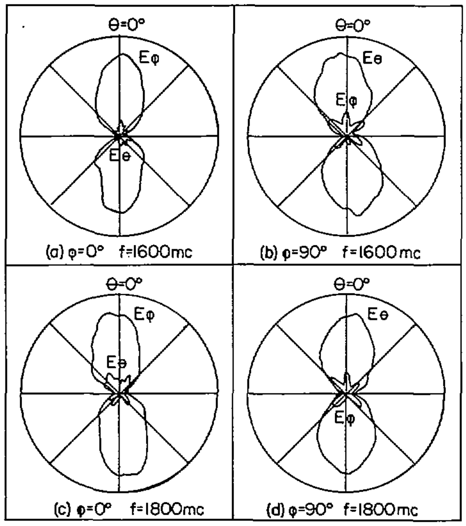

Fig. 14
Radiation patterns $\alpha = \beta = 45^{o}$ $\tau = .50$.

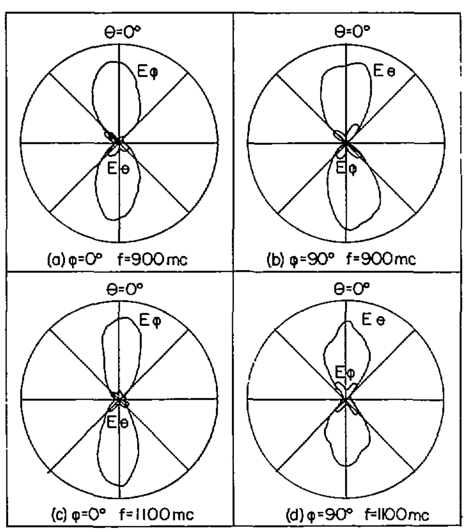

Fig. 15
Radiation patterns $\alpha = \beta = 45^{o}$ $\tau = .25$.

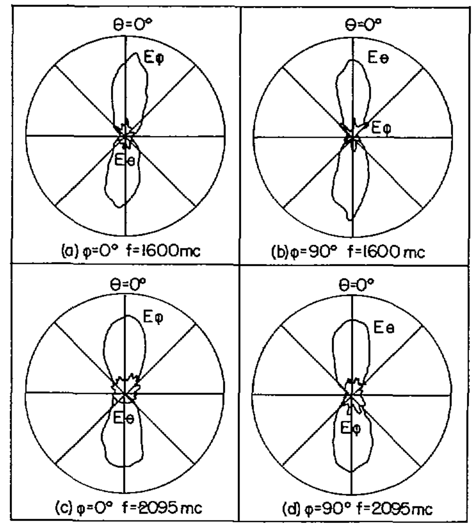

Fig. 16
Radiation patterns $\alpha = \beta = 45^{o}$ $\tau = .25$.

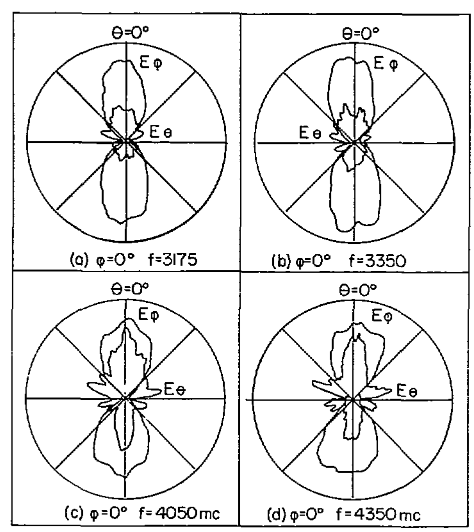

Fig. 17
Radiation patterns $\alpha = \beta = 45^{o}$ $\tau = .15$.