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Determination of Lead-Acid Battery Capacity Via Mathematical Modeling Techniques

Margaret A. Casacca
Student Member, IEEE

Ziyad M. Salameh
Senior Member, IEEE

Department of Electrical Engineering
University of Massachusetts - Lowell
Lowell, Massachusetts 01854

Abstract

The evaluation of the ampere-hour capacity of a lead-acid battery using the technique of mathematical modeling is presented in this paper. The battery model was used to simulated a battery cycle at different temperatures, at different rates of charge and discharge and at different end voltages to determine how the battery parameter of ampere-hour capacity was affected. The parameter obtained from the model simulation was compared with experimental result for verification.

Keywords

Battery, Capacity, Modeling

Introduction

Lead-acid batteries are used to store or deliver energy. Understanding battery behavior is required to properly design electrical systems. Without the proper storage of power a system will not operate efficiently.

The active materials in a lead acid battery are: the positive plates, which are made of lead dioxide (PbO_2); the negative plates, consisting of lead (Pb) and the electrolyte, a solution of sulfuric acid (H_2SO_4) and water (H_2O) as shown in Figure 1.[1] As the battery discharges, both electrodes are converted to lead sulfate. When the battery is charged, this process is reversed.

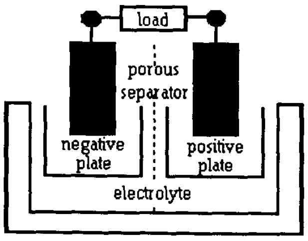

Figure 1: Basic Battery Diagram

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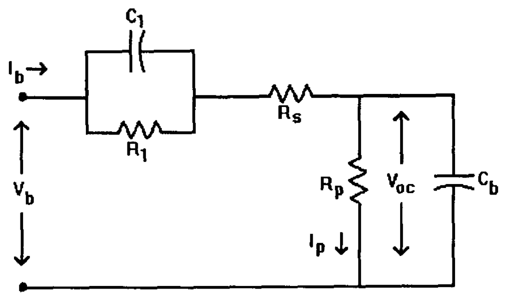
The mathematical battery model, shown in Figure 2, was developed by analyzing experimental test data [2] and manufacturers specifications. Each component represents a nonlinear battery characteristic and is consequentially modeled using exponentials.[3]



C_1 - overvoltage capacitance
C_b - battery capacitance
i_b - current source
i_p - self-discharge current
R_1 - overvoltage resistance
R_p - self-discharge resistance
R_s - internal resistance
V_b - battery voltage
V_{oc} - open circuit voltage

Figure 2: Mathematical Model Equivalent Circuit

Due to the nature of the chemical reactions in the battery, the elements of the battery are modeled with a least-squares approximation of the form:[4]

$$BE = ke^{[wf(V_m - V_{oc})]^{ff}} \tag{1}$$

where:
BE - battery element being modeled
k - gain multiplier
wf - width factor
V_m - mean voltage level
V_{oc} - open circuit voltage
ff - flatness factor

Since the battery elements resemble parabolic functions, a squared exponential was derived (ff = 2). Higher order exponentials could be found but are very sensitive to sudden changes in value and can become unstable.

Functions for four battery elements are required to model a lead-acid battery. These elements are: battery capacitance, internal series resistance, self-discharge resistance and overvoltage.[5]

Changes in temperature affects the magnitude and widths of the self-discharge, and series (internal and overvoltage) resistance functions, therefore compensation is required.[6] By multiplying the non-linear functions by a temperature coefficient temperature compensation is accomplished which takes the form:

$$TC = \left(\frac{R}{R_{ref}}\right)^{\frac{T_{ref}-T}{T_{ref}}} \quad (2)$$

where:
TC - temperature compensated variable
T - temperature of environment
T_{ref} - reference temperature
R - resistance at temperature T
R_{ref} - resistance at temperature T_{ref}

In this paper, the developed mathematical model will be used to evaluate one of the most important battery parameters ampere-hour capacity. Other parameters of the battery include: voltage characteristics, and current characteristics, watt-hour capacity, self-discharge, state of charge. This parameter will be simulated and compared with experimental results for verification.

Model Determination of Battery Capacity

The model simulation was written in the BASIC programming language. The program numerically integrates the current into the capacitors using an iterative sequential solution method to generate the voltage waveforms of the capacitors. Ohms' Law is used to calculate voltage drops across series resistors and current through parallel resistors. Since discharging and charging have different effects on the internal and overvoltage resistances, compensation is provided in this model simulation.

The model simulated a typical battery cycle. A battery cycle consists of five stages: constant current discharge (CCD), discharge idle time (DIT), constant current charge (CCC), constant voltage charge (CVC) and charge idle time (CIT); refer to Figure 3.

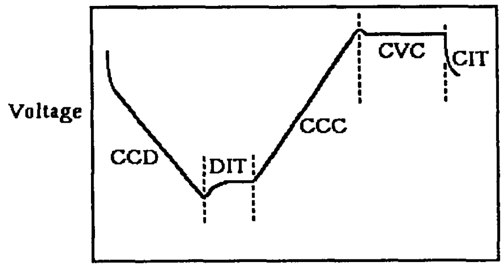


Figure 3: Typical Test Cycle

Ampere-hour capacity is a parameter of the lead-acid battery. Ampere-hour capacity is defined as the number of ampere-hours removed from a battery. This quantity is determined by integrating the rate of current during discharge over time.

Temperature, rate of charge and discharge, and end voltages were varied to determine differences in the battery ampere-hour capacity. The different values that were used in the model simulation are shown in Table 1.

	Warm	Cold
Temperature	35 Celsius	-5 Celsius
High Rate	13 Amps	13 Amps
Low Rate	1 Amps	1 Amps
100% Charged	15.8 Volts	14.2 Volts
75% Charged	14.8 Volts	-----
25% Charged	11.4 Volts	11.65 Volts
0% Charged	10.5 Volts	10.5 Volts

Table 1: Various Model Conditions

Experimental Determination of Battery Capacity

A battery test system was designed, built, and used at the University of Lowell by the photovoltaic program to perform extensive testing of batteries. The battery test system consists of a power supply, a controller rack, a data acquisition system and two temperature-controlled chambers, as shown in Figure 4. Five parameters can be simultaneously varied to analyze the effects of different conditions on battery performance: high end voltage, low end voltage, charge rate, discharge rate and environmental temperature.

Cycle testing was done to verify the results of the model parameters. The tests were performed under the conditions stated in Table 1. During the idle periods specific gravity measurement were taken to insure proper battery performance.

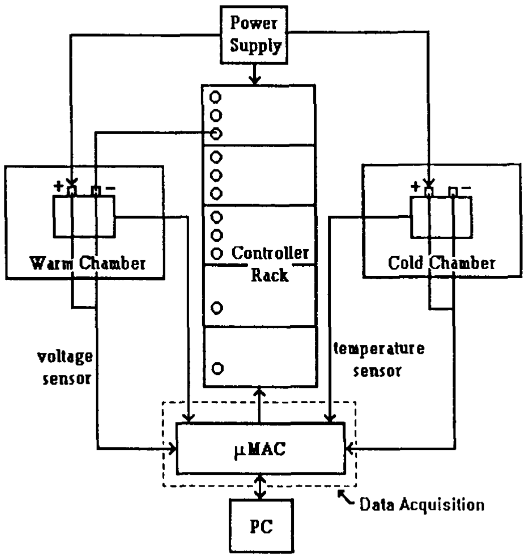


Figure 4: University of Lowell Battery Test System

Discussion of Results

The resulting ampere-hour capacity of the model simulation was compared with experimental data. Figures 5 - 12 show the comparison between the model results (bold line) and the experimental results (thin line). There are three parameters associated with each figure, the rate of charge and discharge in amps, the end points of the cycle test in volts, and the environmental temperature in degrees Celsius.

The capacity produced from the model reflected the experimental capacity in terms of ampere-hours removed and duration of discharge. Table 2 quantitatively displays the ampere-hours removed and returned to the battery.

	Discharge Amp-hours			Charge Amp-hours	
Figure	Model	Exper.	%Error	Model	Exper.
5	113.5	108.4	4.5	119.3	135.2
6	122.0	117.6	3.6	171.8	167.0
7	79	86.1	9.0	119.2	121.8
8	61.0	68.0	11.5	78.9	85.3
9	73.2	66.5	9.2	95.1	99.7
10	93.8	95.8	2.1	132.0	132.2
11	77.6	73.5	5.3	90.6	76.7
12	46.8	47.2	0.9	60.7	57.7

Table 2: Ampere-hour Capacity Comparison

The effect of the various condition on the cycle test is visible in the ampere-hour capacity curves. Changing the rate of charge and discharge, the temperature, and the end voltages had a significant effect on the ampere-hour capacity.

The effect of the temperature variation is shown at a high rate of charge and discharge between Figures 5 and 9 and at a low rate between Figures 6 and 10. In both cases the ampere-hour capacity is decreased as the temperature decreases. It should be noted that the upper voltage in the cold temperature had to be increased to achieve the rated capacity.

Rate of charge and discharge affected the duration of charge and discharge as well as the number of ampere-hour removed and returned. When the rate is increased, the number of ampere-hours is decreased along the duration. This decrease in ampere-hour capacity is found at both the warm temperature by comparing Figures 6 and 5 or Figures 7 and 8, as well as at the cold temperature by comparing Figures 10 and 9 or Figures 11 and 12.

End voltage variation had the effect of decreasing the ampere-hours when the voltage range was decrease. The effect of the higher rate of charge and discharge resulted in a greater loss of ampere-hour capacity than the low rate of charge and discharge. This can be seen at both temperatures. At the warm temperature, the difference between Figures 6 and 7 (low rate) is a loss of 15.7, while the difference between Figures 5 and 8 (high rate) is a loss of 43 ampere-hours. The same results are found at the cold temperature by comparing the difference between Figures 10 and 11 (low rate), with that of Figures 9 and 12 (high rate).

Conclusion

The mathematical model accurately depicts the battery parameter of ampere-hour capacity under various operating conditions. Changes in end voltages, rates of charge and discharge, and temperature did not affect the ability of the battery model to represent the behavior of a lead-acid battery. More over, it was found that the capacity decreased with a decrease in temperature, an increase in the rate of charge and discharge, and by shortening the voltage range. Incorporation of this model in system analysis would allow for accurate and continuous evaluation.

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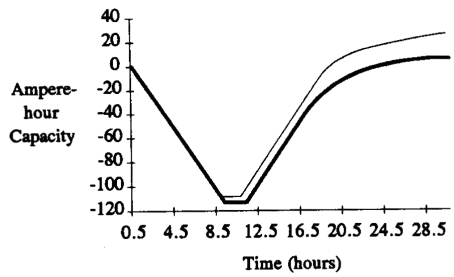

***Figure 5:* Battery Capacity for: 13 A, 10.5-14.2 V, 35°**

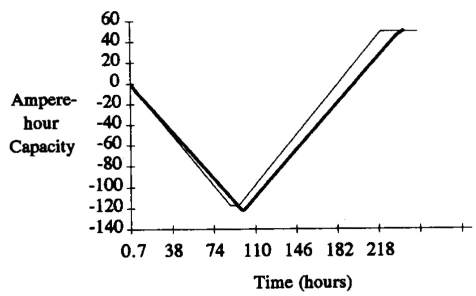

***Figure 6:* Battery Capacity for: 1 A, 10.5-14.2 V, 35°**

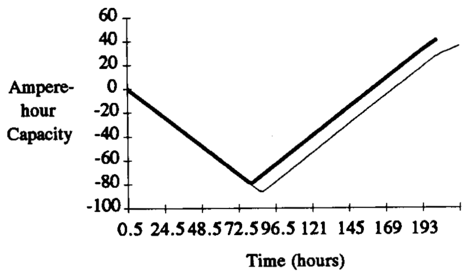

***Figure 7:* Battery Capacity for: 1 A, 11.65-14.2 V, 35°**

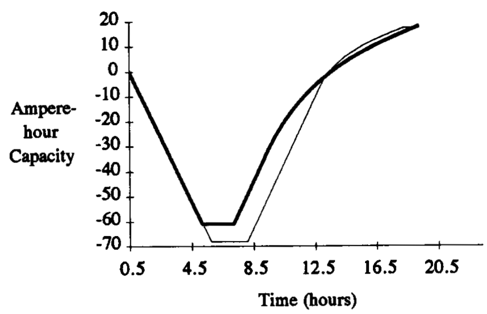

***Figure 8:* Battery Capacity for: 13 A, 11.65-14.2 V, 35°**

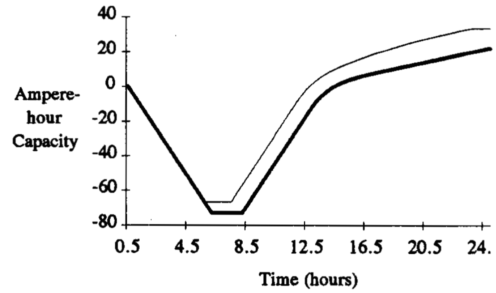

***Figure 9:* Battery Capacity for: 13 A, 10.5-15.8 V, -5°**

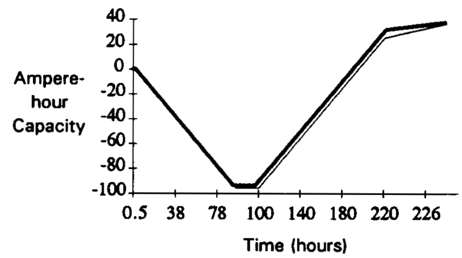

***Figure 10:* Battery Capacity for: 1 A, 10.5-15.8 V, -5°**

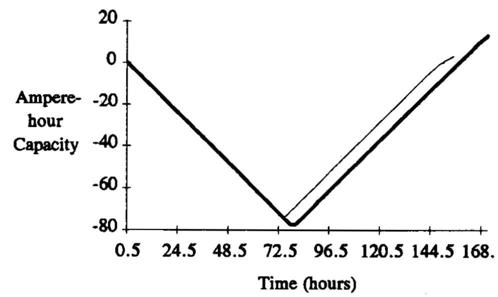

***Figure 11:* Battery Capacity for: 1 A, 11.4-14.8 V, -5°**

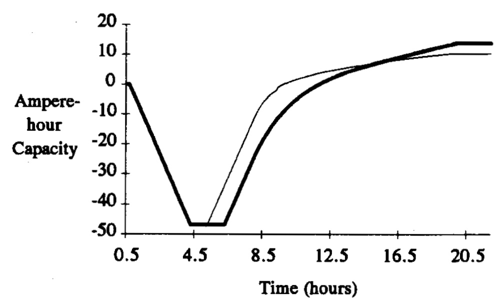

***Figure 12:* Battery Capacity for: 13 A, 11.4-14.8 V, -5°**

References

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Ziyad M. Salameh (S-78, M-82, SM-88) was born in Agraba, Jordan on May 4, 1944. He received his B.S. in Electrical Engineering from Moscow Power Engineering Institute, USSR, in 1974 and his M.S. and Ph.D. from the University of Michigan, Ann Arbor, in 1980 and 1982 respectively. Ziyad Salameh taught at Yarmouk University in Irbid, Jordan from 1982 to 1985. He has taught at the University of Massachusetts -Lowell, Massachusetts since 1985. He is a member of IEEE Power Engineering, Industry Applications and Power Electronics societies. His areas of interest are power electronics, solid state electrical drive systems, modeling of electrical machines, wind and photovoltaic energy conversion systems.

Margaret A Casacca (S-88) was born in Leominster, Massachusetts, U.S.A. on June 3, 1968. She received her B.S. and M.S. Eng. degrees in Electrical Engineering from the University of Lowell, Lowell Massachusetts in 1989 and 1990 respectively. She is currently working on her doctorate at the University of Massachusetts - Lowell. Her areas of interest include mathematical modeling, battery modeling and evaluation, power systems and photovoltaic energy conversion systems.