

# A GENERIC FINGERPRINT IMAGE COMPRESSION TECHNIQUE BASED ON WAVE ATOMS DECOMPOSITION

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## ABSTRACT

Modern fingerprint image compression and reconstruction standards used by the US Federal Bureau of Investigation (FBI) are based upon the popular 9/7 discrete wavelet transform. Multiresolution analysis tools have been successfully applied for fingerprint image compression for more than a decade; we propose a novel fingerprint image compression technique based on recently proposed wave atoms decomposition. Wave atoms decomposition has specifically been designed for enhanced representation of oscillatory patterns to convey temporal and spatial information. Our proposed compression scheme is based upon linear vector quantization of decomposed wave atoms representation of fingerprint images. Later quantized information is encoded with arithmetic entropy scheme. The proposed image compression standard outperforms the FBI fingerprint image compression standard, the wavelet scalar quantization (WSQ). Data mining, law enforcement, border security, and forensic applications can potentially benefit from our proposed compression scheme.

**Index Terms**— Image compression, fingerprints, wave atoms, quantization, arithmetic coding

## 1. INTRODUCTION

The fundamental goal of image compression is to obtain the best possible image quality at an allocated storage capacity. Law enforcement, border security and forensic applications are some crucial fields where fingerprint image compression plays an important role. Emergence of protocols and commercially available products has prompted police forces to use Automated Fingerprint Identification Systems (AFIS) during criminal investigations. The Federal Bureau of Investigation deals with a massive collection of fingerprint database that contains more than 200 million cards and is growing at the rate of 30,000-50,000 new cards daily [1]. The archive consists of inked impressions on paper cards. A single card contains 14 different images: 10 rolled impression of each finger, duplicate (flat) impression of thumb and simultaneous impression of all fingers together. Fingerprint images are digitized at a resolution of 500 pixels per inch with 256 gray levels. Therefore a single fingerprint card requires approximately 10 MB of storage; the investigation of an efficient compression standard, that can significantly reduces the image size while retaining the minutiae (ridges, endings and bifurcations) information, is justified in conjunction with the size of FBI fingerprint database. In addition to the considerable savings in storage capacity, fingerprint image

compression is also desired for effortless archiving and sweeping reduction in transmission bandwidth.

FBI compression standard has incorporated the biorthogonal 9/7 discrete wavelet transform (DWT) filter pair for highly reliable fingerprint compression and reconstruction since 1993. DWT was used due to its capability of space-frequency decomposition of images [2], energy compaction of low frequency sub-bands, and space localization of high frequency sub-bands. Image analysis using DWT is described using a pair of quadrature mirror filter (QMF) and a dual quadrature mirror filter (DQMF). QMF and DQMF are further decomposed into four sets of floating point coefficients:  $h0(Lo\_D)$ ,  $g0(Hi\_D)$ ,  $h1(Lo\_R)$  and  $g1(Hi\_D)$  define the wavelet and scaling functions for each of forward DWT and inverse DWT respectively. Table 1 details the filter coefficients used for image decomposition and reconstruction.

**Table 1:** Filter coefficients for 9/7 biorthogonal FBI compression standard.

Filter Type	Filter Coefficients
h0 (Lo Dec)	[0.03783 -0.02385 -0.11062 0.37740 0.85270 0.37740 -0.11062 -0.02385 0.03783]
g0 (Hi D)	[0.06454 -0.04069 -0.41809 0.78849 -0.41809 -0.04069 0.06454]
h1 (Lo D)	[-0.06454 -0.04069 0.41809 0.78849 0.41809 -0.04069 -0.06454]
g1 (Hi D)	[0.03783 0.02385 -0.11062 -0.37740 0.85270 -0.37740 -0.11062 0.02385 0.03783]

Fingerprint images are decomposed using a 2D DWT which is applied using a separability approach along its rows and columns alternatively resulting into four smaller subsets. These subsets are further decomposed, quantized and coded using different coding techniques. Researchers have proposed various techniques for iterative decomposition such as FBI's 64- subband [3] and Kasaei *et al's* 73-Subband decomposition [4]. Entropy based best basis selection (EBBBS) algorithm [5] has also been proposed for improved sub-band decomposition. Recently proposed fingerprint image compression schemes use genetic algorithm [6-7] to evolve wavelet and scaling numbers for each level of decomposition. In this paper we propose a generic algorithm for fingerprint image compression using wave atoms decomposition. Extensive experiments are performed and our proposed method significantly outperforms the traditional WSQ FBI compression standard.

The remainder of this paper is divided into 4 sections. Section 2 discusses the wave atoms transform along with its implementation details. The proposed method for fingerprint image compression is described in section 3. Experimental results are

discussed in section 4 and section 5 details concluding remarks followed by acknowledgment and references.

## 2. WAVE ATOMS DECOMPOSITION

Fourier series decomposes a periodic function into a sum of simple oscillating functions, namely *sines* and *cosines*. In a Fourier series sparsity is destroyed due to discontinuities (Gibbs Phenomenon) and it requires a large number of terms to reconstruct a discontinuity precisely. Development of new mathematical and computational tools based on multiresolution analysis is a novel concept to overcome limitations of Fourier series. Many fields of contemporary science and technology benefit from multiscale, multiresolution analysis tools for maximum throughput, efficient resource utilization and accurate computations. Multiresolution tools render robust behavior to study information content of images and signals in the presence of noise and uncertainty.

Wavelet transform is a well known multiresolution analysis tool capable of conveying accurate temporal and spatial information. Wavelet transform has been profusely used to address problems in data compression, pattern recognition, image reconstruction and computer vision. Wavelets better represent objects with point singularities in 1D and 2D space but fail to deal with singularities along curves in 2D. Discontinuities in 2D are spatially distributed which leads to extensive interaction between discontinuities and many terms of wavelet expansion. Therefore wavelet representation does not offer sufficient sparseness for image analysis. Following the introduction of wavelet transform, research community has witnessed intense efforts for development of ridgelets [8], contourlets [9], and curvelets [10]. These tools have better directional and decomposition capabilities than wavelets.

Wave atoms are a recent addition to the collection of mathematical transforms for harmonic computational analysis. Wave atoms are a variant of 2D wavelet packets that retain an isotropic aspect ratio. Wave atoms have a sharp frequency localization that cannot be achieved using a filter bank based on wavelet packets and offer a significantly sparser expansion for oscillatory functions than wavelets, curvelets and Gabor atoms. Wave atoms capture coherence of pattern across and along oscillations whereas curvelets capture coherence only along oscillations. Wave atoms precisely interpolate between Gabor atoms [14] (constant support) and directional wavelets [15] (wavelength  $\sim$  diameter) in the sense that the period of oscillations of each wave packet (wavelength) is related to the size of essential support by the parabolic scaling i.e. wavelength  $\sim$  (diameter)<sup>2</sup>.

Two distinct parameters  $\alpha$ ,  $\beta$  represent decomposition and directional ability and are sufficient for indexing all known forms of wave packet architectures namely wavelets, Gabor, ridgelets, curvelets and wave atoms. Wave atoms are defined for  $\alpha=\beta=1/2$  and essential support of wave packet in space (left) and in frequency (right) is shown in Fig. 1.  $\alpha$  indexes the multiscale nature of the transform, from  $\alpha = 0$  (uniform) to  $\alpha = 1$  (dyadic).  $\beta$  measures the wave packet's directional selectivity ( $0$  and  $1$  indicate best and poor selectivity respectively). Wave atoms represent a class of wavelet packets where directionality is sacrificed at the expense of preserving sparsity of oscillatory patterns under smooth diffeomorphisms.

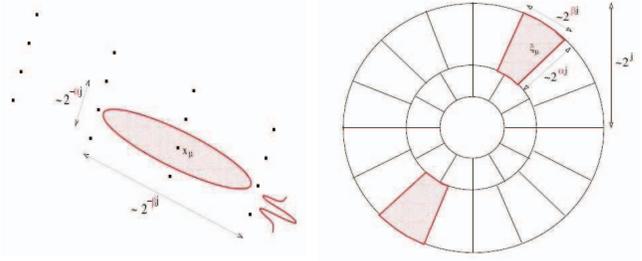


Fig. 1. Wave atoms tiling in space and frequency [12].

### 2.1. 1D discrete wave atoms

Wave atoms are constructed from tensor products of adequately chosen 1D wave packets. Let  $\psi_{m,n}^j(x)$  represent a one-dimensional family of wave packets, where  $j, m \geq 0$ , and  $n \in \mathbf{Z}$ , centered in frequency around  $\pm \omega_{j,m} = \pm \pi 2^j m$ , with  $C_1 2^j \leq m \leq C_2 2^j$  and centered in space around  $x_{j,n} = 2^{-j} n$ . One-dimensional version of the parabolic scaling states that the support of each bump of  $\hat{\psi}_{m,n}^j(\omega)$  is of length  $O(2^j)$  while  $\omega_{j,m} = O(2^{2j})$ . Dyadic dilates and translates of  $\hat{\psi}_m^0$  on the frequency axes are combined and basis function, written as:

$$\psi_{m,n}^j(x) = \psi_m^j(x - 2^{-j} n) = 2^{j/2} \psi_m^0(2^j x - n). \quad (1)$$

The coefficients  $c_{j,m,n}$ , for each wave number  $\omega_{j,m,n}$ , are obtained as a decimated convolution at scale  $2^j$ .

$$c_{j,m,n} = \int \psi_m^j(x - 2^{-j} n) u(x) dx. \quad (2)$$

By Plancherel's theorem,

$$c_{j,m,n} = \int e^{i2^{-j} n \omega} \overline{\hat{\psi}_m^j(\omega)} \hat{u}(\omega) d\omega. \quad (3)$$

If the function  $u$  is discretized at  $x_k = kh$ ,  $h=1/N$ ,  $k=1 \dots N$ , then with a small truncation error (3) is modified as:

$$c_{j,m,n}^D = \sum_{k=2\pi(-N/2+1:N/2)} e^{i2^{-j} nk} \overline{\hat{\psi}_m^j(k)} \hat{u}(k). \quad (4)$$

Since the data is supported inside two disjoint intervals of size  $2^{j+1}\pi$ , symmetric about origin ( $2^{j+1}$  points) instead of an interval of length  $2^j \times 2\pi$ , sum(4) is computed using a reduced inverse FFT inside an interval of size  $2^{j+1}\pi$  centered about origin:

$$c_{j,m,n}^D = \sum_{k=2\pi(-2^j/2+1:2^j/2)} e^{i2^{-j} nk} \sum_{p \in 2\pi\mathbf{Z}} \overline{\hat{\psi}_m^j(k+2^j p)} \hat{u}(k+2^j p). \quad (5)$$

A simple wrapping trick is used for the implementation of discrete wavelet packets and the steps involved are:

1. Perform an FFT of size  $N$  on the samples of  $u(k)$ .
2. For each pair  $(j, m)$ , wrap the product  $\overline{\hat{\psi}_m^j} \hat{u}$  by periodicity inside the interval  $[-2^j\pi, 2^j\pi]$  and perform an inverse FFT of size  $2^j$  to obtain  $c_{j,m,n}^D$ .
3. Repeat step 2 for all pairs  $(j, m)$ .

The overall complexity of the algorithm is  $O(N\text{Log}N)$  and the wavelet packets are decomposed into positive and negative frequency components, represented by

$$\hat{\psi}_{m,n}^j(\omega) = \hat{\psi}_{m,n,+}^j(\omega) + \hat{\psi}_{m,n,-}^j(\omega). \quad (6)$$

Hilbert transform  $H\psi_{m,n}^j$  of eq. (6) represents an orthonormal basis  $L^2(\mathbb{R})$  and is obtained through a linear combination of positive and negative frequency bumps weighted by  $i$  and  $-i$  respectively.

$$H\hat{\psi}_{m,n}^j(\omega) = -i\hat{\psi}_{m,n,+}^j(\omega) + i\hat{\psi}_{m,n,-}^j(\omega). \quad (7)$$

## 2.2. 2D discrete wave atoms

A two-dimensional orthonormal basis function with 4 bumps in frequency plane is formed by individually taking products of 1D wave packets. Mathematical formulation and implementations for 1D case are detailed in the previous section. 2D wave atoms are indexed by  $\mu=(j,m,n)$ , where  $m=(m_1,m_2)$  and  $n=(n_1,n_2)$ . Construction is not a simple tensor product since there is only one scale subscript  $j$ . This is similar to the non-standard or multi-resolution analysis wavelet bases where the point is to enforce same scale in both directions in order to retain an isotropic aspect ratio. Eq. (1) is modified in 2D as:

$$\varphi_{\mu}^+(x_1,x_2) = \psi_{m_1}^j(x_1 - 2^{-j}n_1)\psi_{m_2}^j(x_2 - 2^{-j}n_2). \quad (8)$$

The Fourier transform of (8) is separable and its dual orthonormal basis is defined by Hilbert transformed wavelet packets in (10).

$$\hat{\varphi}_{\mu}^+(\omega_1,\omega_2) = \hat{\psi}_{m_1}^j(\omega_1)e^{-i2^{-j}n_1\omega_1}\hat{\psi}_{m_2}^j(\omega_2)e^{-i2^{-j}n_2\omega_2}. \quad (9)$$

$$\varphi_{\mu}^-(x_1,x_2) = H\psi_{m_1}^j(x_1 - 2^{-j}n_1)H\psi_{m_2}^j(x_2 - 2^{-j}n_2). \quad (10)$$

Combination of (8) and (10) provides basis functions with two bumps in the frequency plane, symmetric with respect to the origin and thus directional wave packets oscillating in a single direction are generated.

$$\varphi_{\mu}^{(1)} = \frac{\varphi_{\mu}^+ + \varphi_{\mu}^-}{2}, \quad \varphi_{\mu}^{(2)} = \frac{\varphi_{\mu}^+ - \varphi_{\mu}^-}{2} \quad (11)$$

$\varphi_{\mu}^{(1)}$  and  $\varphi_{\mu}^{(2)}$  together form the wave atoms frame and are jointly denoted by  $\varphi_{\mu}$ . Wave atoms algorithm is based on the apparent generalization of the 1D wrapping strategy to two dimensions and its complexity is  $O(N^2\text{Log}N)$ .

## 3. PROPOSED IMAGE COMPRESSION STANDARD

Wave atoms decomposition is used for sparse representation of fingerprint images since they belong to a category of images that oscillate smoothly in varying directions. Schematic block diagram of the proposed method is shown in Fig. 2. Discrete 2D wave atoms decomposition is applied on the original image in order to efficiently capture coherence of the fingerprint images along and across the oscillations. An orthonormal basis  $\varphi_{\mu}^+(x)$  ( $\varphi_{\mu}^{(1)} + \varphi_{\mu}^{(2)}$ ) is used instead of a tight frame since each basis function oscillates in two distinct directions instead of one. This orthobasis variant

property is significantly important in applications where redundancy is undesired.

Magnitudes of wave atoms decomposed coefficients, carrying low information content, are either zero or very close to zero hence these can be discarded without a substantial degradation in image quality. An appropriate global threshold is used to achieve desired transmission bit rate. After thresholding significance map matrix and a significant coefficient vector is generated. Significance map is a matrix of binary values that indicates the presence or absence of significant coefficient at specific location. The significance map is divided into non-overlapping blocks of  $4 \times 4$ . These non-overlapping blocks of significance map are vectorized and quantized using a K-means vector quantization scheme with 64 code words. Small blocks of data are used in order to minimize the error during vector quantization. The significant coefficients are quantized using a uniform scalar quantizer with 512 distinct levels.

Quantized significance map and significant coefficients are encoded using an arithmetic encoder. Arithmetic coding is a variable length entropy scheme that attempts to minimize number of bits by converting a string into another representation using more bits for infrequent characters and *vice versa*. As opposed to other entropy encoding techniques that convert the input message into component symbols and replace each symbol with a code word; arithmetic coding represents the entire message into a single number thereby achieving optimal entropy encoding.

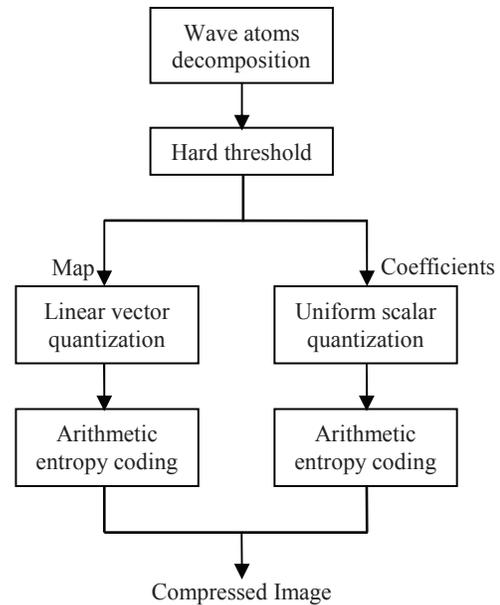


Fig. 2. Block diagram of proposed compression algorithm.

## 4. RESULTS AND DISCUSSION

Various fingerprint images used in FBI's WSQ standard are compressed using the proposed method and substantial improvement in compression is achieved. The quality of various image compression techniques depends upon how close is the reconstructed image to the original image. Different metrics are proposed for investigating the quality of compression algorithms. Some methods investigate similarity while others explore the level of dissimilarity between reconstructed and the reference image. Mean square error (MSE) and peak signal to noise ratio (PSNR)

are two celebrated metrics used to examine the qualitative performance. MSE is a distortion metric that provides a measure of dissimilarity between two images. MSE and PSNR are calculated using (12) and (13) respectively.

$$MSE = \frac{1}{R * C} \sum_{i=1}^R \sum_{j=1}^C |X(i, j) - X^l(i, j)|^2, \quad (12)$$

$$PSNR = 10 \log_{10} \frac{255^2}{MSE}, \quad (13)$$

where  $R$  indicates the number of image rows and  $C$  refers to the number of columns,  $X(,)$  represents the original image whereas  $X^l(,)$  refers to the reconstructed image.

Fig.3 demonstrates a sample fingerprint and the image reconstructed at 0.25 bits per pixel (bpp) using the proposed compression method. From Fig. 3 it is evident that the proposed method using wave atoms decomposition does an excellent job in preserving the fine details in a fingerprint image i.e. the minutiae (ridges ending and bifurcations) at lower bit rates. Table 2 compares the PSNR obtained using our proposed method with the FBI's WSQ compression standard at varying bitrates. As shown in Table 2 fingerprint compression based on wave atoms decomposition produces a significant improvement in PSNR at high compression ratios (low bit rates) in comparison with FBI's WSQ fingerprint compression standard.

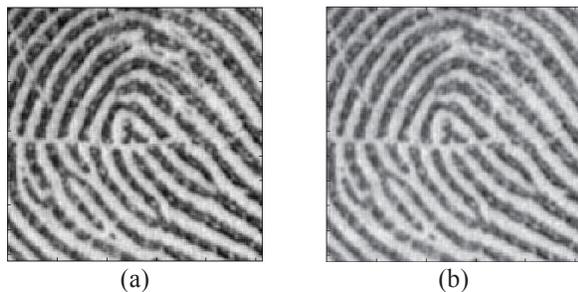


Fig. 3. (a) Original and (b) compressed image at 0.25 bpp.

Table 2: Bit rate vs. PSNR for proposed method and FBI's WSQ.

Bit rate (bits per pixel)	FBI's WSQ [3] PSNR (db)	Proposed method PSNR (db)
0.1	23.72	28.62
0.2	26.25	31.21
0.3	27.96	31.68
0.4	29.36	32.42
0.5	30.37	32.65

## 5. CONCLUSION

Application of wave atoms based decomposition for fingerprint image compression results into a significant improvement in PSNR compared to FBI's WSQ fingerprint compression standard. The improvements in PSNR are more pronounced and distinct at lower bit rates and validate the fact that wave atoms multiresolution analysis offers significantly sparser expansion, for oscillatory functions, than other fixed standard representations like wavelets, curvelets and Gabor atoms and captures coherence of pattern both across and along oscillations. Law enforcement, multimedia, and

data mining related applications can benefit from our proposed compression scheme.

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